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MATHEMATICAL MODELING OF FACILITY MAINTENANCE PLANNING.

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Mathematical Modeling of Facility Maintenance Planning

A thesis submitted in partial satisfaction of the requirements for the degree Master of Science in Engineering

by

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ABSTRACT OF THE THESIS

Mathematical Modeling of Facility
Maintenance Planning

by

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Professor J. Morley English, Chairman

The planning of maintenance for large facility complexes as found in the military and in large corporations requires a rational decision making process for efficiently allocating resources for maintenance. Engineering economics has provided a basis for choosing between competing projects where all values are reducible to economic values. When the number of alternative projects becomes large, the search for optimal combinations of projects becomes difficult. Decisions must also consider value parameters, intangibles, not represented in economic evaluation. It is the purpose of this thesis investigation to contribute to the development of a mathematical model for planning the maintenance of large facility complexes.

In order to accomplish the foregoing purpose, the relation of facility maintenance planning to the larger system it serves was analyzed. A general planning model was developed, based upon a value system design process. The general planning model was found to require extensive additional research with respect to the definition and analysis of values in facilities maintenance other than those having economic interpretations. A simplified planning model based on economic value only, is defined. Mathematical techniques including linear, integer and dynamic programming used in capital budgeting were investigated for applicability to the planning problem.

A computer solution to the planning model objective function employing dynamic programming was evaluated. The dynamic programming model was found to be tractable for large sets of projects when a decomposition technique was employed, but impractical because of the lack of constraints on the solution, making it incompatible with current financial management practices.

CHAPTER I

INTRODUCTION

1.1 Objectives

The overall objective of this thesis is to develop a mathematical model for planning the maintenance of facilities for organizations operating large scale facility complexes. Of special concern is the military base system. The specific objectives are: to define the problem of planning facility maintenance for large scale facility systems and its relationships with the objectives of the overall system; to develop a planning system model by applying systems engineering value design methodology; to report on mathematical techniques used for analyzing capital investments showing their relation to the facility problem; and to develop and briefly evaluate an algorithmic solution to a simplified maintenance planning model.

1.2. Outline of the Facility Maintenance Planning Problem

The management of facilities maintenance may be defined to have two principle elements. First, it must decide what must be done, and second how the tasks can be accomplished in the most efficient manner. The latter problem is one involving: organization of the work force, planning work schedules for efficient production, and

supervising the performance of the work. This task is extremely important and is generally treated under such categories as production planning and control. Great improvements have been made in this field aided by more effective building products and production equipment.

The task of deciding what must be done remains in the domain of the top level decision maker, aided by a planning organization. He is faced with the task of deciding specifically what must be maintained, and to what extent it is to be maintained. The simple answer is to maintain everything in a "like new" condition. Unfortunately, the resources available seldom permit such decisions.

The simple solution of maintaining everything in a new condition would not be seriously in error, if the original requirements for the facilities remained constant (also assuming the new condition was the minimum acceptable condition). The real world condition is, however, one of constant change. New requirements for facilities are generated and old ones eliminated. It is essential that maintenance of a facility be accomplished to the extent that the facility will continue to be required.

Another form of the problem is deciding which of several deserving maintenance tasks will be performed first, in a limited budget where each is equivalent

in cost. The question of relative values has now been introduced. The maintenance planner must decide which project, if performed, will add the greatest benefit to the overall system which the set of facilities serves.

These decisions are difficult to make and often sufficient information is not at hand to properly judge between competing maintenance projects. The same problem faces the maintenance planner whether he is responsible for a single facility, a military base or the total system of military bases.

There is a need for a maintenance planning system which permits the planner at each echelon to contribute the information he is best qualified to provide, considering his vantage point. The system should permit all of the values pertinent to good resource investment decisions to be expressed; it should permit the maintenance planner to apply mathematical techniques to search for the optimal plan for investing the limited resources available to the set of maintenance requirements; and finally the planning system's introduction should be gradual and compatible with the existing method of solving the planning problem.

1.3 The Approach

A mathematical model for planning the maintenance of facilities should consist of these principal parts:

1. Definition of the facilities maintenance problem and its relation to the overall system it serves.
2. Development of a value system to be used to judge the merits of all proposed allocation plans.
3. Selection of an algorithmic technique to search the set of feasible solutions for optimal solutions.

Facilities maintenance planning has been defined as a management subsystem serving the total set of mission subsystems oriented toward accomplishment of the overall system objectives. The national defense system has been used as the specific case under consideration. A general value system is developed which has sufficient dimension to include all the pertinent value laden parameters of the general planning problem.

Because of the lack of practical measures of values other than economic, a simplified value system based upon the economic theory of value is then derived which may be suitable as an initial step in applying the more general value system. Finally, a dynamic programming formulation is presented which may be used to search for optimal solutions to the resource allocation problem. A computer program was developed to evaluate its feasibility.

1.4 Related Work Done by Others

While no specific works were found in the literature of facility maintenance planning, the following works provide the background upon which the developments in this thesis are based. The general field of engineering economics has been the source of decision rules applied in selecting mutually exclusive plans where economic measures dominate. Grant and Ireson (9) provide one of the basic works in general engineering economics, while Barish (1) and Morris (15) are more current works attempting to introduce probabilistic considerations into economic decisions.

The economic theories of decision making have been limited in their ability to measure all the pertinent value parameters in a decision making situation; and it has been necessary to develop broader measures of value and to find means to compare dissimilar value measures. Systems engineering has been a source of value system design. Hall (10) presents a treatment of value measurement including both the economic and the psychological theories of value. Fields (7) and Fox (8) provide treatments on cost effectiveness. A general system design value model is developed by Lifson. His work is the basis for the general planning model proposed in Chapter III. The methodology of systems engineering design has been employed in this thesis to structure the maintenance planning problem, because it is systems

oriented and requires a value model for comparing solutions analogous to the comparison of alternative design concepts.

Capital budgeting treats the problem of choosing between multiple courses of action to provide the greatest economic rewards for the allocation of resources. Models have been developed based upon the economic theory of value in the fields of securities investments (13), capital improvement project selection (14), and equipment replacement (20). Capital budgeting problems are resolved to mathematical models requiring optimization of an objective function subject to constraint. The methods used in capital budgeting have served as a guide to the selection of an algorithm for resolving the objective function developed in Chapter III.

The mathematical methods of linear, integer and dynamic programming have been developed in capital budgeting models. Linear programming advanced by Dantzig (5) was applied to the capital budgeting problem by Weingartner (22). Dynamic programming advanced first by Bellman was applied to the capital budgeting problem by Weingartner (21) and Cord (4).

1.5 Order of Presentation

Chapter I provides an introduction to the problem background, relative to the general development of this

thesis.

Chapter II provides basic definitions to be used and defines further the relation of facility maintenance to the larger system in which it is imbedded. The mission versus facility life relation is also treated.

Chapter III provides a development of a general value system design process and its application to the maintenance planning problem. A simplified planning model based upon economic theory only is also developed.

Chapter IV reviews the mathematical techniques of linear, integer and dynamic programming as applied to capital budgeting problems.

Chapter V presents the development of a dynamic programming algorithm for solution of the objective function of the economic planning model developed in Chapter III and reports the results of computer runs employing the algorithm.

Chapter VI presents conclusions.

CHAPTER II

FACILITIES MAINTENANCE PLANNING--DEFINITIONS, SYSTEM ORIENTATION AND A COMPARISON OF EXPECTED MISSION AND FACILITY LIFE

This thesis is concerned with the planning of facilities which are elements of large complexes of facilities designed to satisfy dynamic sets of mission requirements. Of special interest is the problem of planning the maintenance for the set of military bases which form a part of our national defense system. While the emphasis will be placed upon the military problem, the presentation is applicable to other organizations, especially governmental, which operate facility maintenance programs independent of programs for capital improvements to their respective physical plants.

In this chapter terms to be used will be defined. A system orientation of the facilities planning problem will be presented, and the relation between mission and facility life, as it affects the planning problem will be discussed. The purpose of this chapter is to prepare for the development of a mathematical model of a value oriented planning system to be presented in Chapter III, and the investigation of techniques for optimizing the objective function to be derived as presented in Chapters IV and V.

2.1 Terminology

Facility

A facility is a structure or ground structure including all of the attachments and equipment that serve to create a desired environmental state for the accomplishment of a mission or set of missions. It excludes equipment that can be detached or removed from the facility which is productive in nature, e.g., a machine tool may be detached from a building, a locomotive may be removed from a section of railroad track.

System

A system is a set of objects with relationships between the objects and their attributes. Objects are simply the parts or components of a system. Attributes are properties of objects. For example, springs exhibit spring tension and displacement. Relationships tie the system together.[1]

Subsystems

A system which is an element or object within a larger system is called a subsystem. This concept gives rise to the hierarchial order of systems, wherein all subsystems may be defined as being an object in successively larger systems. The largest system is defined as the universe.

New Construction

The provision of a facility or expansion of an existing facility for the purpose of meeting the requirements

of a new mission, or expanded mission or set of missions. It includes the rebuilding of a facility for a new mission.

Missions and Objectives

A general statement of need is a problem situation for which a system is to be designed and operated. The statement of missions and objectives is the basis for the design of a value system for the evaluation of possible solutions to the problem.

Maintenance

Maintenance is defined as all work necessary to keep a facility in an operable condition for the satisfactory performance of its assigned missions. Maintenance is defined to include two subdivisions:

1. Routine maintenance

   The frequent or continuous work performed on a facility to keep it at a satisfactory operational level of condition. Routine has the connotation of being sets of small independent tasks performed repeatedly which do not require significant replacement of component parts. (In Chapter III this definition will be modified to include operational expenses incurred because of the deterioration of the facility.)

2. Repair

   Repair is the infrequent work performed on a facility to return a facility to a satisfactory

operational level of condition. This work includes the replacement of constituent parts of the facility. It includes no expansion of the capacity of a facility and is not performed for the purpose of rebuilding an unused and deteriorated facility for the satisfaction of a new mission.

2.2 A Systems Orientation For Facilities Maintenance

The definitions of systems and subsystems permit any system to be defined as a subsystem of some larger, encompassing system. A key consideration of systems design is the optimization of the objectives of the system. Care must be exercised in optimization efforts, for the relations between the system under consideration and adjacent subsystems in its encompassing system, may result in a suboptimization in the larger system. It is important to know how the system under consideration affects its environment. Hitch and McKean (12), advise that the effect of subsystem optimization on at least one level higher in the hierarchy of systems should be examined to insure that the subsystem optimization is desirable. The hierarchy of systems in which the maintenance planning for military facilities is imbedded will be reviewed.

The largest system which can reasonably be visualized as defining the objectives of military facilities maintenance is the national government. The policies esta-

blished at this level state the need for the next lower system, the national defense system, represented by the Department of Defense. It is within the defense system that the objectives of the facilities maintenance subsystem are defined. A recent address by the Assistant Secretary of Defense (Comptroller), Robert N. Anthony,[2] outlined the current set of systems employed by the Department of Defense to define its missions and objectives. The largest set of subsystems are called Major Programs. Figure 1 lists the set of major subsystems.

FIGURE 1

MAJOR DEFENSE PROGRAMS

1. Strategic Forces
2. General Purpose Forces
3. Specialized Activities (includes MAP[3])
4. Airlift and Sealift
5. Guard and Reserve Forces
6. Research and Development
7. Logistics
8. Personnel Support
9. Administration

The major problems are subdivided into Program Elements. For example, B-52 Squadrons and Base Operations (Offensive) are two elements under the major program strategic forces. Both of the above subdivisions may be

described as mission oriented, that is, they define activities which must be performed to satisy the principle objectives of the defense system--to counter possible hostile action against the nation.

Figure 2 is a list of Functional Categories which are defined as the elements of the new program elements defined above. At this level a new systems orientation is introduced--the management system. Each of the functional categories is common to a degree to each of the program elements (mission oriented). This permits or requires that a management system be designed which will provide the services of the specific function to all of the major programs and program elements. The system of functional categories may be considered a shadow system designed to optimize the performance of each functional category, according to a set of management objectives under a set of constraints defined by the major programs and program elements.

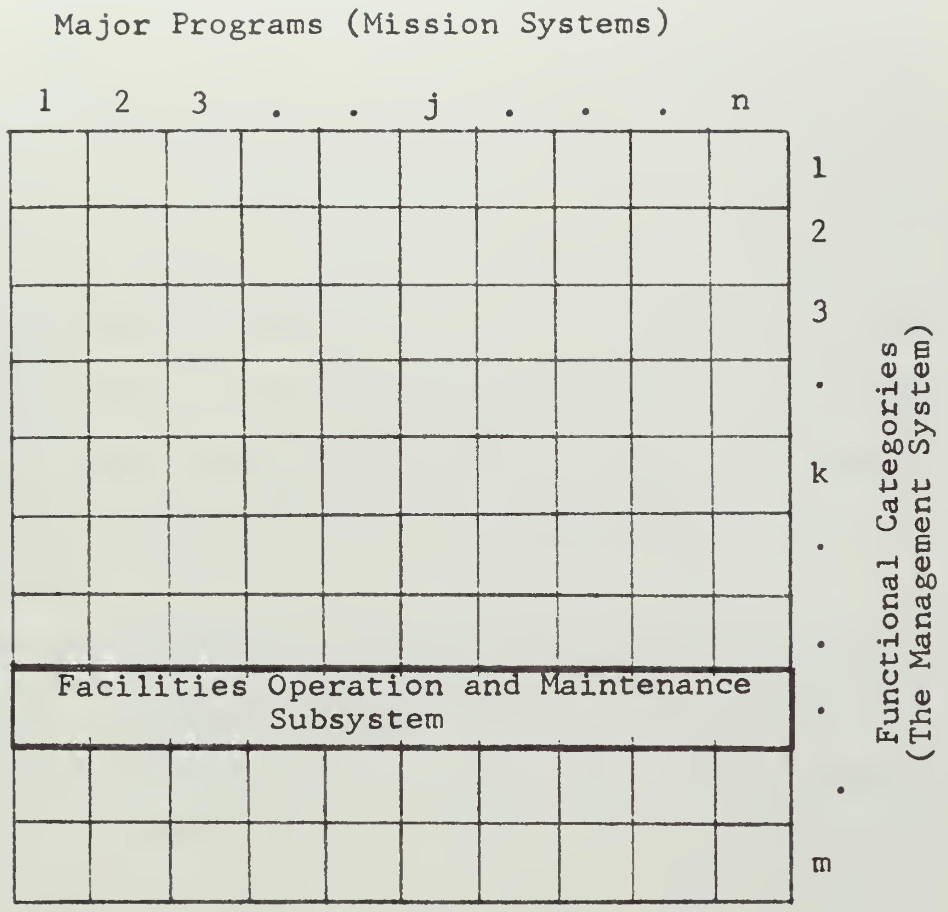
Figure 3 illustrates the relationship between the major programs and the two functional categories which are relevent to this thesis: Operations and Maintenance of Utilities; and Maintenance of Real Property Facilities. A maintenance planning system will be developed to optimize the objectives of the management system of the two functional categories within the constraints imposed by the operational system. The two functional categories

FUNCTIONAL CATEGORIES

OF THE MILITARY MISSIONS SYSTEM

1. Mission Operations
2. Supply Operations
3. Maintenance of Materiel
4. Modernization
5. Transportation
6. Communications
7. Medical Operations
8. Food Service
9. Personnel Housing Operations
10. Overseas Dependent Education
11. Other Personnel Support
12. Base Services
13. Operation and Maintenance of Utilities
14. Maintenance of Real Property Facilities
15. Minor Construction
16. Administration

FIGURE 2



INTERRELATIONS BETWEEN THE MILITARY MISSIONS SYSTEMS, MAJOR PROGRAMS AND MANAGEMENT SYSTEM, FUNCTIONAL CATEGORIES

Figure 3

will be referred to jointly as, facilities maintenance.

2.3 Expected Mission and Facilities Life

By limiting the problem to facilities maintenance, the planning of new facilities is placed beyond the scope of this thesis. However, realistic maintenance planning must reflect the dynamic character of the set of missions and objectives of the defense system, and in this respect, the problem is similar to that of planning new construction. Maintenance resources must not be applied to facilities which have no future mission assigned or foreseen. The more difficult case is to decide how much maintenance is justified for a facility with a limited estimate of mission life.

A problem inherent in facilities planning is the requirement to make an initial investment in a facility which can not be consumed by an originally assigned mission. This is not to say that all facilities are subject to the assignment of short duration missions, but that some facilities are, and optimal planning requires that the problem be considered. If successive missions could be defined by the mission oriented planning system, the problem would be resolved. The life as a measure of the total requirement for a facility would be defined, and maintenance plans prepared accordingly. In the absence of long range mission projections which may be taken as deterministic, the life of a facility must be taken to be

a function of the expressed mission life, and the characteristics of the facility which indicate that it may be expected to serve future undefined missions.

Two methods of examining future missions probabilities present themselves. The first is based upon an evaluation of individual facilities. For example, an office building is usually of such a general design that it can be expected to be usable for its designed purpose for the duration of successive missions until its economic life is reached.

The extreme of this case is the special purpose facility designed for the performance of a single research experiment. Such a facility may be so specialized that it has no future use. Both of these cases are simple to analyze, as stated; but add to the first case a consideration such as remoteness of location, and to the second, an extension of mission life to include a series of experiments of unknown duration and the problems become complex.

The key consideration in both of the above cases is the desired longevity of the maintenance work performed; not whether or not the facility should be maintained for some currently defined mission. This first method, then, is an evaluation of each individual facility, attempting to determine the probability that it will have some estimated life beyond the currently planned major program life.

The second evaluation is of complete base complexes.

We have recognized that when missions are defined, facilities are constructed to meet these requirements, and that because of the dynamic nature of the defense system, requirements are often eliminated, leaving an excess of facilities. The total military base complex system must be analyzed to eliminate excess capacity. Regardless of the value of an individual facility, it may be declared excess as a part of an entire military base.

To evaluate the probability of the life of a facility beyond the currently defined major program life, requires an evaluation of the individual facility for convertibility to meet future mission requirements, and an evaluation of the total set of military base complexes. The estimation of facility life is an important element in maintenance planning. In the following chapters it will be assumed that a reasonable estimate can be made.

Footnotes:

1. Definition paraphrased from Hall, page 60.
2. Address given October 24, 1966 before the financial Management Roundtable, a Department of Defense Conference.
3. MAP is the Military Assistance Program for foreign countries.

CHAPTER III

DEVELOPMENT OF A VALUE SYSTEM FOR FACILITIES MAINTENANCE PLANNING

In Chapter II the relations between facilities, their maintenance, and the higher order systems in which they are imbedded as a subsystem were established. It was also established that management of the maintenance of facilities is a subsystem of a total management system for functional elements of major programs oriented toward accomplishment of missions. In this chapter a planning concept for maintenance of a complete system of facilities will be developed by application of value system design methodology to the facilities problem. The value system design process will be described initially, and then the steps of the process will be applied to the problem.

The purpose of using the value system design process is to provide a well defined model to which refinements may be added as additional definitions of value in the problem are developed through research. This presupposes the hypothesis that management and planning for a facilities system is analogous to: establishment of a value system; and by some process synthesizing and evaluating all feasible courses of action to select that course of action which will produce the highest value

to the system. Application of the general value system planning model is limited by the practical limits on defining elements having value, referred to as intangibles, and the inability to mathematically solve the problem even were all values completely defined. Assumptions will be made which reduce the problem to a more tractable form.

3.1 The General Value System Design Process

The design of a system, whether it be a piece of hardware, such as an airplane, or a management process, must have some criteria by which the goodness of the design may be judged. Therefore, a value system design process is essential to systems engineering methodology. Hall (10) distinguishes two value theories: the economic theory, and the psychological theory. The economic theory is the type of engineering economics presented by Grant and Ireson (9), Barish (1), and others. The psychological theory is defined to contain such value measuring techniques as the Von Neumann and Morgenstern Utilities Theory, and the Churchman and Ackoff (3) Order Scales, among others.

The combination of the two general theories is an objective in systems engineering and operations research value systems. Cost effectiveness is an example. Lifson (23) presents a general value system design process, a modified form of which will be presented here. The steps

of the process are shown in Figure 4[1], and will be treated in the following paragraphs.

Problem Formulation

To initiate the process, a general statement of the problem must be formulated. In a product design process, an initial statement of the problem is provided by the customer, but in the case of designing a system within an organization, higher level management must provide the initial statement of the problem.

Estimate of Needs and Objectives $\{N\}$

In this step specific needs, missions or objectives which are to be fulfilled by the system are defined. English (6) describes design as an iterative process and indicates that even the initial statement of needs may not be accurately stated, but will be improved as the process evolves. $\{N\}$ is the set of needs goals and objectives which are to be accomplished by the system.

Estimate of Resources $\{R\}$

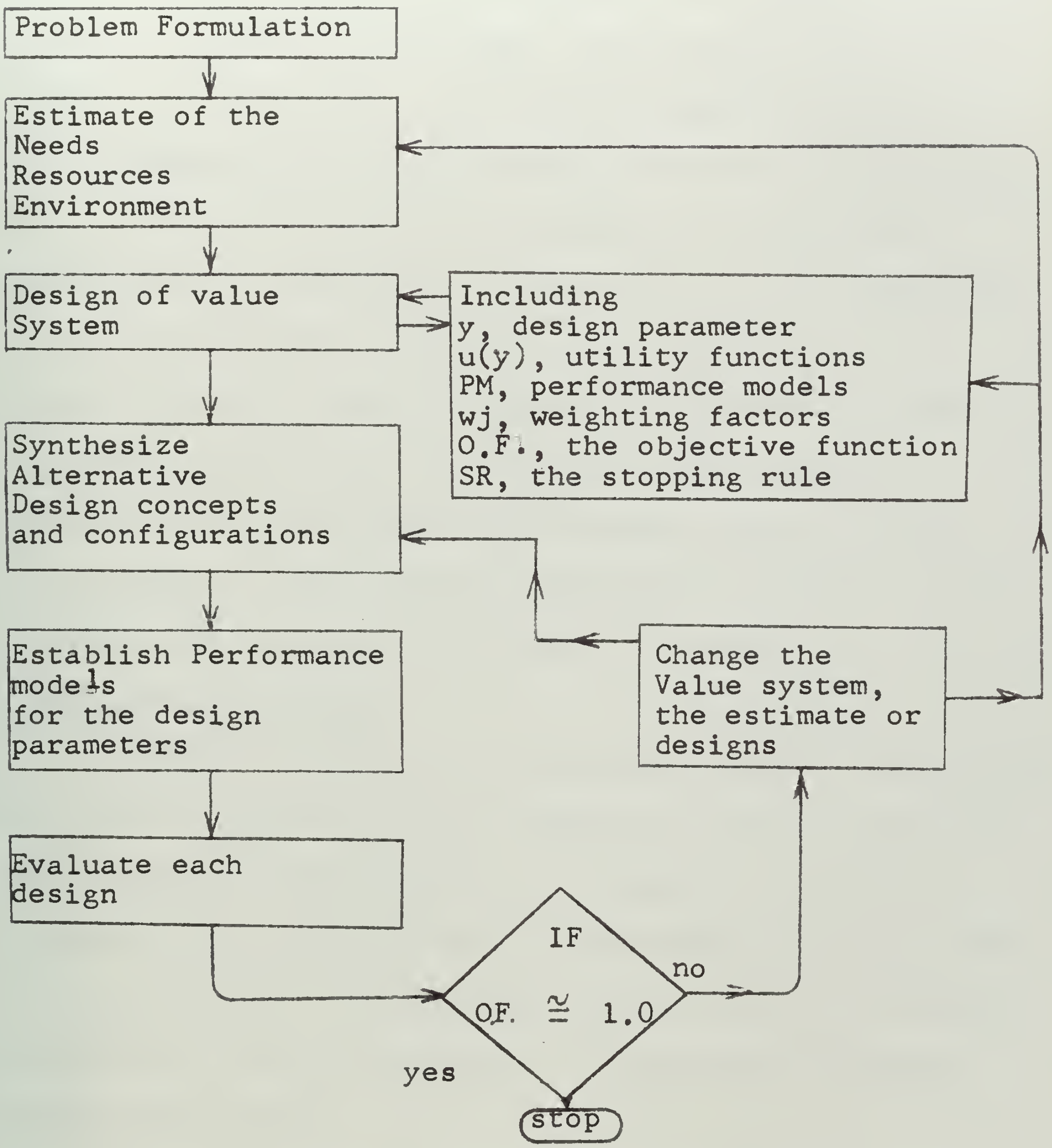
The resources pertinent and available to the system for fulfilling the needs $\{N\}$ must be defined. Quantitative representations are desirable. The statement of resources will be used as a set of constraints on the solution of the system. $\{R\}$ is the set of vectors which describe available resources.

Estimate of the Environment $\{\bar{E}\}$

The system must exist in an environment which is

FIGURE 4

THE VALUE SYSTEM DESIGN PROCESS



some larger system. For the purposes of design, the statement of environment may be limited to those elements which are pertinent to the system. For example, environmental elements may be present which have little or no effect on the system. Such elements may be omitted. In the general case each element of the set of pertinent environments may consist of a vector of subordinate environmental elements. An assumption in the definition of the environment is that the decisions made in the system are of an order of magnitude sufficiently small that the state of the environment is not affected. $\bar{E}$ is the set of pertinent environments, each of which is a vector of environmental elements not significantly affected by the decisions within the system.

Design Parameters $\{y_j\}$

A design parameter is a criterion which is considered to have an influence on the degree to which the needs $\{N\}$ of the system are satisfied. An example of a design parameter in facilities may be the number of square feet of usable space in a building. $\{y_j\}$ is the set of design parameters which are selected to represent the system.

Utility Functions

Utility functions are the relations which convert the quantitative measures of a design parameter, $\{y_j\}$, into measures which express value to the satisfaction of the system needs. Schlaifer (1) presents methods

of constructing utility functions. Utility functions to be used in the value system are normalized utilities. That is, they represent the segments of the scale of a utility function defined by the aspiration points and the point of indifference, where the indifference point is assigned the value of zero utiles and the aspiration points may be assigned the value of plus and minus one. The negative aspirations may not exist. Normalized utilities have the properties of linear functions.[2]

<u>Performance Models</u>

Performance models are the relationships which transfer the characteristics or properties of a given design alternative into the quantitative measure of a design parameter, $\{y_j\}$. For example a design parameter may be the floor loading capacity of a building. . .

$$y_j = f\left[\{I\}, \{E_e\}, \{S\}, \{W_t\}, \{S_m\}, \{D_l\}\right] \qquad 3\text{-}1$$

Where $\{I\}$ = the set of moments of inertia of beams and columns in the structure

$\{E_e\}$ = the set of moduli of elasticity for the beams and columns in the structure

$\{S\}$ = the set of beam and column lengths in the structure

$\{W_t\}$ = the set of possible wind conditions

$\{S_m\}$ = the set of possible seismic loads

$\{D_l\}$ = the set of dead loads on the structure

In this example the performance model is a structural analysis of the building, where wind and seismic loads are given in the pertinent environmental vector, $\left|\bar{E}_k\right|$.

<u>Design Concepts</u>

A design concept is a possible solution to the system which is to be evaluated by the value system. It is described by a set of vectors $\left|\bar{X}_k\right|$ for the k<u>th</u> design concept. $\left|f(y_j\}_L\, k\right|$ is a set of explicit probabilistic effectiveness estimates associated with a configuration, $\bar{X}_k$, of a design concept. Each $f(y_j\}_L\, k$ is a probability density function of design parameter y_j, given that $\bar{X}_k$ is implemented in the L<u>th</u> environment. The probability of the occurrance of the environmental states is defined by the set $\left|P_L\right|$ of discrete probabilities of occurances of the L<u>th</u> environmental state.

<u>The Utility Matrix</u>

The elements of the value system may be visualized as a three dimension array as shown in figure 5. Each of the $u_{jk\,L}$ functions is defined to be a normalized utility value.

<u>Probability Matrix</u>

The probability matrix for a given design configuration shows the probability functions associated with given design parameters and environmental conditions (figure 6). Each $f_{jk\,L}$ is the probability that for a given design parameter, y_j, and a given state, L, a

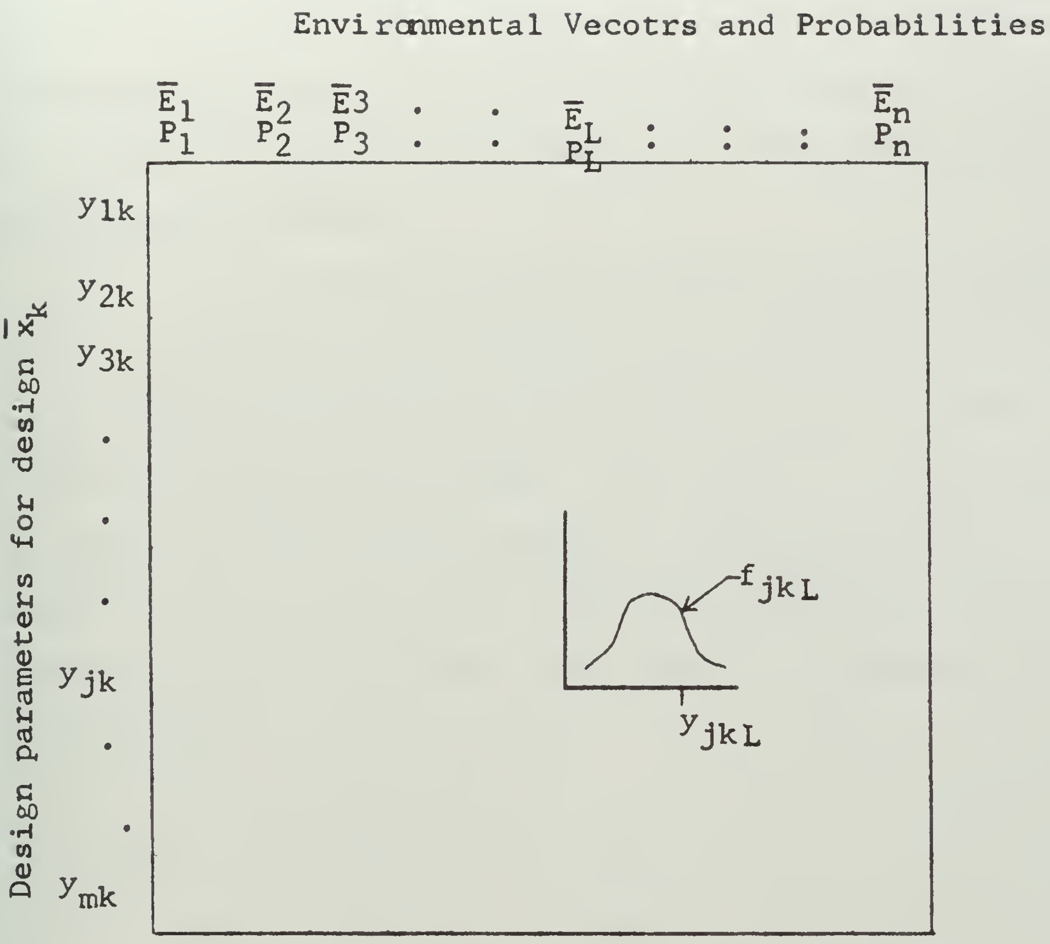
Environmental vectors and probabilities

Design parameters

$$
\begin{array}{c|cccccccccc|}
 & \bar{E}_1 & \bar{E}_2 & \bar{E}_3 & \cdot & \cdot & \cdot & \bar{E}_L & \cdot & \cdot & \bar{E}_n \\
 & P_1 & P_2 & P_3 & \cdot & \cdot & \cdot & P_L & \cdot & \cdot & P_n \\
\hline
y_1 & u_{11} & u_{12} & u_{13} & \cdot & \cdot & \cdot & u_{1L} & \cdot & \cdot & u_{1n} \\
y_2 & u_{21} & & & & & & \cdot & & & \cdot \\
y_3 & u_{31} & & & & & & \cdot & & & \cdot \\
\cdot & \cdot & & & & & & \cdot & & & \cdot \\
\cdot & \cdot & & & & & & \cdot & & & \cdot \\
y_j & u_{j1} & \cdot & \cdot & \cdot & \cdot & \cdot & u_{jL} & & & \cdot \\
\cdot & \cdot & & & & & & \cdot & & & \cdot \\
\cdot & \cdot & & & & & & \cdot & & & \cdot \\
\cdot & \cdot & & & & & & \cdot & & & \cdot \\
y_m & u_{m1} & \cdot & \cdot & \cdot & \cdot & \cdot & u_{mL} & \cdot & \cdot & u_{mn} \\
\hline
\end{array}
$$

MATRIX OF UTILITIES--NO DESIGN SPECIFIED

Figure 5



MATRIX OF PROBABILITY DENSITY FUNCTIONS: GIVEN DESIGN CONCEPT $\bar{x}_k$ FOR VALUES DESIGN PARAMETERS, y_{jk_L}

Figure 6

value, y, of the design parameter, y_{jkL}, will occur.

Weighting Factors w_j

Weighting factors are a measure of the importance attached to a given design parameter to the satisfaction of the set of needs, N. Weighting factors may be developed using Churchman, Ackoff and Arnoff order scales (3).

The Objective Function

The value of elements of alternative design concepts can be evaluated by the parameters, y_j, presented above. It is necessary to develop a relation which describes the total value of the system. One method is to develop one single measure of the system by operating on the elemental values of a design concept represented by the design parameter utilities. The general expression may be stated as:

$$\text{Objective Function}_k = f(w_j, u_{jL}, f_{jL}, P_L) \qquad 3\text{-}2$$

Where $j = 1,2,3. . .m$ and $L = 1,2,3. . . n$

The value of the objective function of the k<u>th</u> design concept is a function of the weighting factor, w_j, for design parameter, j, the normalized utility, u_{jL}, of design parameter, j, for environmental state, L, the probability, f_{jkL}, that the value, y_{jkL}, of the design parameter j, will occur, given environmental state L, and the probability that the environmental state L will occur for the set of all combinations as j takes on values from 1 to m

and L takes on values from 1 to n. The functional relationship is not defined in the general case, but an additive function which assumes linearity, permitted by the normalized utility functions is assumed as a simplication. Equation (3-2) may then be restated as:

$$O.F. = \sum_{j=1}^{m} \sum_{l=1}^{n} (w_j u_{jL} f_{jL} P_L) \qquad 3\text{-}3$$

The above form of the objective function assumes that all elements of the value system can be translated into a single unit of measure. If this can not be done, a multidimensional objective function results. The cost effectiveness model is a two dimensional value system used in systems design.

Synthesis of Alternative Designs

The general value system has been established and may be used to evaluate a set of competing design concepts to be synthesized in this step. Design of the performance model usually must follow the synthesis of alternative designs, because the translation of design properties to the dimensions of the design parameters may be unique for each design.

Evaluation of Each Design and the Stopping Rule

Application of the value system to each design concept will result in an objective function value for each design. A decision must then be made to accept or reject

the design having the highest objective function value. If the decision to reject is made, a review of each element of the value system, as well as the set of design concepts is in order. During the design process, additional information may have been acquired which may change one or perhaps all of the value system relations. After revisions have been made, the value system is applied, repeatedly, until an acceptable design is selected.

3.2 Facilities Maintenance Planning Value System

The steps of the value system design process will be applied to facilities maintenance, to define the planning problem, and to provide a model in which the psychological theory of value may be applied when measures of these values are developed. A simplified objective function based upon the economic theory of value is developed which will be used in Chapters IV and V in the application of techniques for finding optimal design concepts, S_k.

3.21 Statement of Needs $\{N\}$

The primary objective of the facility maintenance system is the satisfaction of requirements for facilities established by the missions assigned by the major program system. Maintenance represents the work necessary to keep a facility functioning, but a characteristic of some facility maintenance is its postponability. A

facility may function for a period of time with no maintenance, but a point is reached where work must be accomplished. Inherent in this characteristic is the question of degree of mission satisfaction. Assume that there is a range of facility conditions which will satisfy a mission requirement. At the highest level of condition, an optimal value of mission satisfaction may be defined and at the lowest level of condition mission performance may be marginal, or the effects are not measurable quantitatively. An objective of facilities maintenance must be to maintain a level of facility condition for all facilities in the system, which produces the optimal value to the combined missions oriented system, and facilities maintenance system. Smith (18) uses the term concrescence, meaning growing together, to define the process where the conflicts of two intersecting value systems are resolved by adopting a single value system encompassing both of the original systems. The principle of concrescences requires that the maintenance planning system have as an objective the continuous provision of the facilities optimally required by the mission oriented system. A second objective of the maintenance system is that a minimum of resources be expended. The conflict must be resolved by defining negative values to non-attainment of the optimal facility conditions. The cost of operating a primary mission due to less than optimal mainten-

ance may be taken as a cost of postponement of maintenance.

3.22 Statement of Resources |R|

The set of resources at the disposal of the maintenance planner include money, manpower, and material. Money represents the dominant resource under conditions of stable national economy. The amount of money that can be made available is a decision made at higher levels within the defense system and should reflect the relative need for facilities, compared to other elements of the functional categories. The constraints on money are presented in the form of a fiscal budget.

Manpower is defined as the "in house" capability to perform work. It consists of the military and civilian personnel who are assigned to facility maintenance tasks. Manpower planning is an additional tool in controlling the expenditure of money and therefore must be regarded as a constrained resource in most cases. Man power planning serves two purposes: first, it fosters stability of employment, thus, serving national objectives above the national defense system; and second, it forms a ready tool for managing, in the event that money fails as a planning tool for allocation of national resources.

Material is a resource that is usually constrained by the timing of its availability. When the economy is in a relatively stable state, material is unconstrained, but during a military emergency, this resource becomes

heavily constrained as normal production processes convert to military production. A maintenance planning model should consider constraints on the above resources.

3.23 Environments $\{N\}$

Two general statements of environmental states may apply to military facility maintenance planning:

1. four states--peace, preparing for war, war, post war, or
2. the set of all feasible combinations of missions for all combinations of facilities.

Both must be considered approximations of reality, but neither are practical. The first is indefinite, and the second can not be defined. If it were, the solution would be intractable.

The assumption to be made in the elementary mathematical model is that only a single environment exists which is the best estimate available of the future. This assumption can be made for all of the characteristics of the problem. The probabilistic representation of some elements such as life of requirement may be permitted and still retain a tractable solution in small problems. In making this assumption about a dynamic system, it is necessary to apply static analysis frequently or as often as new projects and resource conditions present themselves.

3.24 Design Parameters $\{y\}$

The set of design parameters must measure how well a maintenance plan achieves the two primary objectives: optimal condition of facilities for the set of assigned missions, and minimum consumption of resources. To reflect reality the design parameters must measure both psychological and economic values inherent in each design concept. This has been and remains a principle limitation to effective modelling of reality, for little work has been done from a facilities viewpoint to establish a relation between the two value systems. The approach has been to assign economic value to all recognizable quantities of value and to treat the remaining psychological values as non quantifiable intangibles treated subjectively along with the economic analysis when making decisions. The result often seems to be an arbitrary decision, when the decision maker, influenced by the intangible factors decides counter to the economic facts. It would be more accurate to say, assuming that the decision maker is correct, that the economic analysis failed to reflect the reality of the case.

The development of a comprehensive set of pertinent design parameters for the complete evaluation of the facilities plan is beyond the scope of this thesis. It will be suggested that the following are some of the value laden parameters which require definition and quantification for inclusion in the facility maintenance

planning value system:

1. Habitability of personnel support facilities.
2. Esthetic qualities of interiors and exteriors of facilities and adjacent landscapes.
3. Conveniences created for personnel who operate the mission, but have no contracted claim for conveniences improved or provided by higher maintenance of a facility.
4. Safety.
5. Maintenance of facilities for possible future missions.

This list is far from exhaustive, but serves only to illustrate the nature of values which are not treated by the economic value theory. Items 4 and 5 do have economic interpretations, if values can be assigned to future requirements for the facilities. In item 4 the loss of a human life or the cost of crippling a human being, must be determined. English[3] treated an analogous case concerning the seismic design of buildings. An economic value of an earthquake caused failure was taken to be a function of the probability of failure of the building for conditions of load and failure causing load. The economic value, then, is the expected value derived from the sum of the probabilities of failure and the associated cost of failure for each value of load.

For the mathematical model treated in Chapter V,

a simplication will be made by resorting to economic value theory and the engineering economics approach. Traditional engineering economics as presented by Grant and Ireson (9) and Barish (1) use the parameters of net present worth of future cost streams, equivalent annual cost and internal rate of return as measures of values in engineering projects. The three may be briefly described, as follows:

1. Net present worth, $P_1 = \sum_{i=1}^{n} c_i \frac{1}{(1+r)^i}$ 3-4

Where c_i = the net annual cost for time period i,

r = a discount rate for the decreased values of future money,

$\frac{1}{(1+r)^i}$ = the discount factor,

and n = the number of time periods during the life of the project.

2. Equivalent annual cost, R

$$R = P \frac{i(1+r)^n}{(1+r)^n-1} \qquad 3\text{-}5$$

3. Internal rate of return

The internal rate of return is found by solving equation (3-4) for the value of r, given that the present worth is equal to zero.

If all of the attributes affecting the condition of a facility can be translated into annual costs, one of the above parameters can be used to measure the value of a facility maintenance plan. For the purposes of

this thesis, net present worth money saved, S, will be used for measuring the positive effects of investing resources in facilities maintenance. A second non additive design parameter, present worth money spent, Q, will be used for measuring the negative value of investing resources in facilities maintenance.

Objective Function

The cost effectiveness type two dimensional objective function will be used. Effectiveness is defined to be the net present worth money saved, S, and cost, Q, the net present worth of money spent to create the effectiveness condition.

$$\text{O. F.} = f(S,Q) \qquad 3\text{-}6$$

3.25 Alternative Design Concepts X_k

The alternative courses of action in the planning sense represent the set of different feasible combinations for allocating resources to the total facility system. It is postulated that the state of the system is to be measured against a set of optimal facility conditions. Several additional assumptions will now be made to provide a total measure of system condition:

1. If the system missions remain constant, there is some cost, which if allocated, will maintain the system at its current state of condition indefinitely. (This condition is not necessarily the optimal.)

2. The constant condition cost can be divided into two general types of cost: Routine Maintenance, (Definition from Chapter II modified to include: all operational inconveniences which have an economic interpretation or which reflect the extent of variance from the optimal conditions defined at the beginning of the assumed constant condition period), and repairs (Chapter II definition.)
3. That repair projects can be defined which estimate the routine maintenance costs of a facility for its expected mission life.
4. That routine maintenance costs will change when when a facility is repaired, providing reduced routine maintenance cost.
5. That the set of all projects for the repair of the total set of facilities represents the possible future condition states of the facility system as a function of the quantity of resources to be applied.
6. That the maintenance plan will only include repair projects which influence, but do not control, the management of the routine maintenance budget. (No attempt will be made to manage routine maintenance in this planning system.)

The alternative design concepts $\overline{X}_k$, courses of action, consist of the total possible combinations for allocating resources to the set of repair projects over the total budget planning horizon for k = 1,2,3. . . o.

3.26 Performance Models

First Design Parameter, S.

a. $S_k = f(R_i, c_{ij}, c'_{ij}, r_i)$ 3-7

b. $$S_k = \left[\sum_{j=1}^{m} c_{ij} \frac{1}{(1+r)^j} - \sum_{j=1}^{L_i} (c_{ij}) \frac{1}{(1+r)^j} + R_i \frac{1}{(1+r)^L} + \sum_{j=1}^{n} c'_{ij} \frac{1}{(1+r_i)^j} \right]$$

Where i = project identification number, i=1,2, 3. . .m

k = the number of the design concept, k=1,2,3. . .o

R_i= the cost to repair project i

c_{ij}= the cost of routine maintenance of project i in the jth time period before the repair is funded, j=1,2, 3. . .n

n = the total number of budget periods in the life of each project or some arbitrary limit in which all costs beyond the limit are discounted back to the final included time period, n. (Permits an infinite future.)

c'_{ij}=the cost of maintenance for the ith project after the accomplishment of the repair project.

r_i = the rate of disutility for value

$\frac{1}{(1+r_i)^j}$ = the time discount factor of present worth value for the ith project for values in the jth time period.

L_i= the time period in which the repair for the ith project is made.

The net present worth return, S, for the course of action $\overline{X}_k$ consists of the sum of the returns from the set of repair projects, i, through m, where the return from each project is the sum of the discounted future cost stream for the life of the project, minus the total discounted revised cost stream of the project if the repair were funded in year L=1.

The revised cost stream is the set of original period costs before the repair is made, plus the cost of the repair, plus the set of periodic costs which occur after the repair project is funded to the end of the facility life.

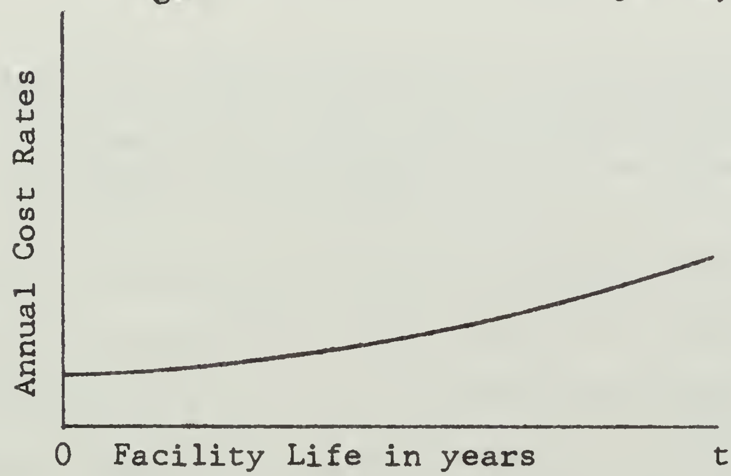
Figure 7a shows an example of a cost stream before a repair has been made for a single project. Figure 7b shows the revised cost stream which will occur if the project were funded immediately. Figure 7c shows a combination of Figures a and b for the case where L is the year that the project is funded.

The two arbitrary limits introduced into the problem are the budget horizon, and an arbitrary time period during which costs are considered. The budget horizon becomes a practical limit over which the maintenance

a.

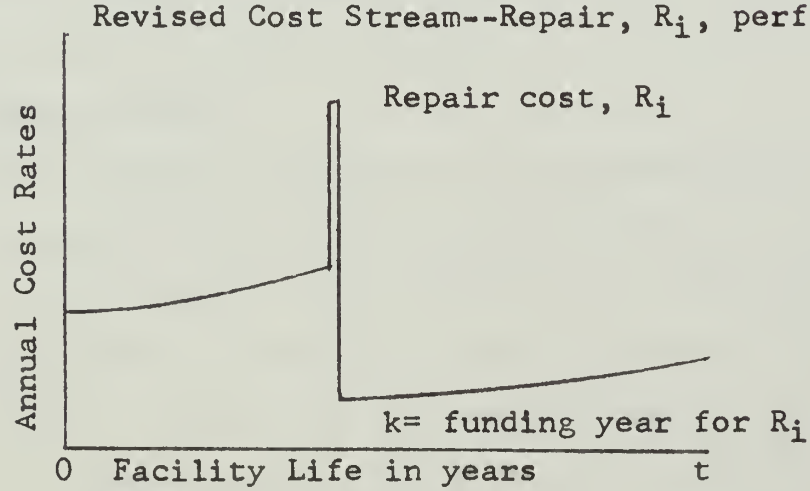
Original Cost Stream--No Repairs, R_i, Perf.

b.



Revised Cost Stream--Repair, R_i, perf.at t=0

c.



Revised Cost Stream, Repair, R_i, perf.at t=k

PROJECT COST STREAMS

Figure 7

plan is to be valid. The effect of this horizon on the selection of projects is one of the elements investigated in Chapter V. The arbitrary maximum life of a project of n years presents difficulties where the actual life exceeds this period. The total cost stream, before the repair can be represented as follows:

$$S_{ik} = \left[c_1a_1 + c_2a_2 + c_ja_j. \; . \; .c_na_n + a_n \sum_{f=1}^{\infty} c_fa_f \right] \qquad 3\text{-}8$$

> Where the last expression on the right represents the discounted sum of all future costs beyond the time period discounted to time period n and then to the present, a_j is the time discount factor for period j.

The difficulty occurs in projecting revised cost streams beyond period n because the final term of a revised cost stream depends upon the time of funding the repair. This limitation may be reduced by making n large to reduce the significance of errors.

<u>Discrete Budget Values</u>

A final practical feature of the representation of the cost stream is the discrete representation of periodic costs in lieu of a continuous fromulation. This permits the project planner to represent cost estimates in the format of fiscal budgets and to express expected jumps in the cost stream caused to large routine maintenance costs that occur infrequently. The latter comment is counter to the defined meaning of repair and routine

maintenance previously stated. Therefore, the definition of routine maintenance will be extended to include repairs which do not exceed some prescribed level of cost. All repairs exceeding this amount must be included in the form of repair projects.

3.26 Second Design Parameter, Q

The second design parameter, Q, present worth cost of resources is:

$$Q_k = \sum_{i=1}^{n} R_{iL} \frac{1}{(1+r_i)^L} \qquad 3\text{-}9$$

Where L is the year each repair R_i is funded.

3.27 Evaluation of Alternative Design Concepts (Courses of Action)

The final step in the process is to evaluate each feasible course of action and select the one which has the most desirable objective function values. The two dimensional objective function has been defined as a cost effectiveness model, because it relates an effectiveness defined as a net present worth savings, S_k, to a present worth cost of resources, Q_k. The method of evaluating the objective function is the subject of Chapters IV and V. The problem to this point has been defined in a form analogous to capital budgeting models. In Chapter IV the characteristics of mathematical programming techniques for capital budgeting models will

be reviewed to find a suitable model for evaluating the facilities maintenance planning system objective function. In Chapter V a dynamic programming algorithm is developed for the solution of the objective function and evaluations of its use pertinent to the practical application of the algorithm are described.

Footnotes:

1. Flow chart and symbology suggested by Dr. R. B. Andrews.

2. See Lifson for a description of normalized utilities.

3. Unpublished paper "Economics of Structural Safety in Seismic Design" by J. Morley English.

CHAPTER IV

OPTIMAL VALUE SEEKING MATHEMATICAL TECHNIQUES FOR CAPITAL BUDGETING PROBLEMS

In Chapter III a value system for planning the maintenance of facilities was developed. The value system establishes the set of all possible courses of action which must be evaluated by application of the objective function, equation (3-6). In this chapter mathematical techniques which have been applied to other capital investment problems are reviewed. The purpose of this review is to show the advantages and disadvantages of the several techniques based on the work done by others, and to show the reasons for the choice of a dynamic programming technique for solution of equation (3-6). The three general methods are linear programming, integer programming and dynamic programming.

4.1 Linear Programming Basic Formulation

Dantzig (5) has been the principle contributer to the extension of linear programming to a wide variety of problems suitable for computer solutions. Included is a large body on allocation models. Weingartner (22) has contributed extensively to its application to the capital budgeting problem. The capital budgeting problem may be described simply as finding a set of investments (I_s) from a larger set I, $I_s \in I$, such that the

return from the allocation of a sum of money over a number of time periods is maximized. The problem is constrained to spend no more than the amount allocated to each time period. This may be stated:

a. $\text{O. F.} = \text{Maximum} \sum_{j=1}^{n} b_j x_j$ 4-1

b. $\text{subject to} \sum_{j=1}^{n} c_{bj} \leq C_t \qquad t = 1,2,3, \ldots, T$

c. $0 \leq x_j \leq 1 \qquad j = 1,2,3, \ldots, n$

b_j = the net present value of the jth project discounted to the present

c_{tj} = the outlay required for the jth project in the ith time period

C_t = the maximum permissible outlay in the tth period

x_j = the fraction of the jth project accepted

The objective function for equation (4-1a) represents the sum of the returns expected when the set of optimal fractions x_j of the total set of projects is found by solution of the linear programming algorithm. The set of equations (4-1b) constrains the outlay in each of the budget periods t to the set of constant values C_t. When each project may be funded only once, equation (4-1c) constrains the solution to cases when the maximum investment on any project is the value of its maximum or total outlay. The basic linear programming model

has these characteristics:

1. It permits constraints on budgets to be set in each of a number of budget periods.
2. It permits the incremental funding of a project over a set of budget periods, not always consecutive.
3. Fractional projects may occur.
4. The optimal time for initiating the funding of a project is not considered.

4.11 Additional Feasible Constraints

The limitation of the basic linear programming formulation and of most traditional forms of multiple analysis is the assumption of complete independence between projects (21). This assumption was also made for the repair projects defined in Chapter III. Linear programming permits the consideration of three idealized types of interdependencies between projects: mutual exclusion, contingency relations, and forced acceptance.

Mutually Exclusive Projects

In equipment replacement problems, mutually exclusive projects may be illustrated by the following example. A new machine tool is to be purchased, where there are several competing models available. Each model has an associated return value and a resource cost. The acceptance of one model, excludes the acceptance of any

of the other models available. In the context of facility repairs, two or more alternative repair methods may serve to solve a facility deficiency. Each method may vary in its investment cost, expected durability, or other value oriented characteristic. Acceptance of one method excludes the others. The linear programming constraint which provides this condition may be stated:

$$\sum_{j \epsilon y} x_j \leq 1 \qquad 4\text{-}2$$

Where j is the set of mutually exclusive projects

Equation (4-2) requires that no more than one of the projects in the mutually exclusive set of j be accepted. A limitation to the linear programming algorithm is the possibility of accepting more than one fractional project whose sum is less than one.

4.12 Contingent Projects

It may be desirable in the facility repair problem to divide a repair undertaking for a large facility into several increments, each of which serves a useful purpose, independent of the undertaking of the others, but accomplishment of certain minor projects should be funded after the major ones. This condition defines the minor projects as projects whose funding is contingent upon the prior or concurrent funding of the major projects.

Another form of this example is the case where the second project is of no value unless a specified initial project is funded. These examples imply a time relation. An example which can be included in the linear programming model is illustrated by a case where the desirability of purchasing a machine tool attachment is contingent upon the purchase of the basic machine. In the linear programming model consider the case where project a must be funded before project b. This may be insured by requiring the following additional constraints:

a. $x_b \leq x_a$ 4-3

b. $x_a \leq 1$

If $x_a = 1$, that is, project a is accepted, then project b may be accepted or rejected. The non-negativity con-constraint of equation (4-1c) requires x_b to be at least zero. The addition of equations (4-3 a and b) requires project b to have a value between zero and one. Again the problem of fractional projects prohibits a complete contingency constraint.

4.13 Mutually Exclusive and Contingent Projects

The conditions of the previous two forms of interdependencies may be combined. Consider the facilities repair case, where either one of two types of roof repairs must be performed on a building. If one of the

roof repair projects is accomplished, the remodelling of the interior may proceed. Either project a or b must be funded, before project c. The linear programming constraint may be stated:

a. $x_a + x_b \leqq 1$ 4-4

b. $x_c \leqq x_a + x_b$

<u>4.14 Contingent Project Chains</u>

The basic contingent constraint may be extended to require a specific sequence of funding projects:

a. $x_a \leqq 1$ 4-5

b. $x_b \leqq x_a$

c. $x_c \leqq x_b$

Thus, project <u>a</u> must be funded before project <u>b</u>, and project <u>b</u> must be funded before project <u>c</u>.

<u>4.15 Forced Acceptance of Projects</u>

The forced acceptance of projects may be treated as external to the application of linear programming by merely deceeasing the quantities, C_t, available for outlays in the respective budget periods by the amount of the one or more projects for which funding is demanded.

<u>4.2 Integer Programming</u>

The integer programming formulation is stated in the same manner as the linear programming model with the additional constraint that all x_j be integral. This additional constraint eliminates fractional projects which may occur in the linear programming model. The limitation on the model is the performance of the available integer programming algorithm. Weingartner (21) reported that in a problem of this type using three constraints and 10 projects, conversion was not obtained within 5000 iterations.

4.3 Dynamic Programming

4.31 The Basic Recursion Relation

The basic allocation problem can be formulated in the recursion relation of dynamic programming. The linear programming problem of equation (4-1) is analogous to the knapsack problem which Bellman and Dreyfus treat using dynamic programming (2). Instead of constraints on the total volume and total weight, the usual case in the knapsack problem, these constraints are replaced by constraints on outlays in budget periods. Theoretically, the number of constraints of this nature may be greater than two, but computational limitations place limits on the total number of constraints feasible. Dynamic programming is based upon the "Principle of Optimality) stated by Bellman (2).

> "<u>The Principle of Optimality</u>. An optimal policy has the property that whatever the initial state and initial decisions are, the remaining decisions must consitutte an optimal policy with regard to the state resulting from the first decision."

A proof of this principle is also presented by Bellman (2). The Principle Of Optimality permits the statement of a general recursion relation as follows:

$$f_n(q) = \left| \text{Maximum } g_n(q_n) + f_{n-1}(q-q_n) \right| \qquad 4\text{-}6$$
$$0 \leqq q \leqq Q \qquad 0 \leqq q_n \leqq q$$

Where $f_n(q)$ = maximum return feasible when a quantity of resource q, the state variable is allocated to the n stages of the problem.

The right side of equation (4-6) provides that f_n will be maximized for the investment of a quantity of resource q when the allocation of q is divided between the return g_n from the n<u>th</u> stage and f_{n-1} all previous stages, such that their sum is maximized. This required that the return from g_n + f_{n-1} be evaluated for all values of q as it varies over all feasible values from zero to q. The Principle of Optimality prohibits negative resource allocations. The projects in the allocation or capital budgeting problem are treated as stages in the recursion relation, equation (4-6) but no particular order for adding projects is necessary.

<u>4.32 Multiple State Variables and Dimensionality</u>

Equation (4-6) may be extended to the multiple budget case by adding for each budget time period t one additional state variable. The linear programming formulation, equation 4-1 may be converted to the recursion relation:

a. $f_n(C'_1, C'_2, C'_3, \ldots, C'_t)$ 4-7

$$= \text{Max}\left|bx_n + f_{n-1}(C'_1 - c_i x_i,\ C'_2 - c_2 x_i,\ \ldots, C'_t - c_{tn} x_n)\right|$$

$i = 1, 2, \ldots, n$

$t = 1, 2, \ldots, t$

subject to:

b. $C'_t - c_{ti} \geqq 0$

c. $f_o(C') = 0$

Where $C'_1 C'_2 \ldots C'$ are the budgets for the t time periods

i = identifies the project (all n projects have been considered.)

f_n $(C'_1, C'_2, \ldots, C'_t)$=the maximum return from an optimal allocation of funds from the set of C'_t budget periods to the n projects

Each C'_t is a state varaable of the dynamic programming formulation. Inthe solution to the above type problem where only the discrete valuse zero or one are permit= ted for the fractional projects, x_i, funded, all of the feasible combinations of values of the set of c_t for each stage of the problem must be evaluated and the sets of decisions made, recorded, or stored in a computer

solution.

While the computational time using a computer is important to obtaining tractable solutions, the critical consideration is currently the demands made upon rapid memory capacity for storing the decisions from the previous stages. Nemhauser (16) provides the following equation for estimating computer emeory capacity:

$$\text{Rapid Memory Storage Units Required} = 3NK^P \qquad 4\text{-}8$$

Where N = the number of stages (projects)

P = the number of state variables per stage

and K = the number of feasible values per project

As an example and comparison of current computer capacity, consider an allocation problem where ten projects which may have two feasible values each, either zero or one and the number of budget periods, that is state variables, is ten. Then $N = 10$, $K = 2$, and $P = 10$, and rapid memory storage requirements is approximately 30,000 words of computer memory. By comparison the IBM 7094 computer has a 32,000 word rapid access memory. Even for this small problem, this model computer approaches full capacity. Emphasis is placed upon computer rapid memory, because the speed of solution of the extensive enumerations required for discrete value problems is primarily a function of data access time. If external

memory units, such as magnetic tape and discs are used, the access time is on the order of hundreds or thousands of times slower than the main rapid access memory of the computer.

In equation (4-8) the increase in the number of state variables causes a power increase in the storage capacity requirements. Bellman terms this problem the "Curse of Dimensionality." As stated above, the dynamic programming formulation of capital budgeting problems of equation (4-1) present no theoretical problem, but its algorithmic solution for the discrete value case is limited by the capacity of the computing equipment available.

4.33 Interdependence of Projects

The types of interdependence of projects described under linear programming may also be included in the dynamic programming model. However, each equation in the constraint formulation must be treated as a state variable in the recursion relationship. The dimensionality of the problem creates a practical limit to the number of dependent relationships which can be treated.

4.34 Application to Capital Budgeting

Both Cord (4) and Weingartener (21) have reported application of dynamic programming to capital budgeting. Cord used a two state variable formulation using the

Lagrange multiplier, suggested by Bellman (2) to reduce dimensionality. His recursion relation was of the form:

$$f_n(I') = \underset{\substack{0 \leq I' \leq I \\ 0 \leq x_n \leq 1}}{\text{Max}} \left[P_n x_n - q w_n x_n + f_{n-1}(I' - I_n x_n) \right] \qquad 4\text{-}9$$

Where I = funds available for allocation to capital projects

I_i = the funds required by the ith capital project where $i = 1, 2, \ldots, N$. The I_i's are constants

P_i = the expected annual income over the life of the investment foom the ith capital investment

q = the Lagrange multiplier

x_i = a variable constrained to one of two values: zero, if the ith investment is not included in the budget; one, if the ith investment is included in the budget.

v_i = the variance of the expected interest rate of return on the ith capital project.

w_i = $(I_i v_i / I)$ The ith variance weighted by the ratio of the ith investment to the total funds available.

Cord's formulation includes a constraint on the total variance assigned to the selected set of projects, in addition to the constraint on total funds available. The function of the variance constraint is to limit the risk to the investor. Hillier (11) presents probabilistic relations for the investment problem, and Markowitz (13) English (6), and Morris (15) describe the use of variance

for the measurement of risk in investments as used by Cord. Using the IBM 7070 computer, Cord solved a 25 project problem in 12 minutes. Weingartener questions the validity of the argument that the introduction of a probability distribution can automatically be regarded as treating the problems of uncertainty. His complaint is not with the derivation of the probabilistic information as presented by Hillier, but the means by which the data is obtained.

Weingartner (21) reports on the development of a dynamic programming computer program using a program language similar to FORTRAN with the exception that the results of the strategies are stored and computed in binary form. This permits the program to test 2000 strategies at each stage with 10 separate constraints. For Cord's problem, 63 seconds were required for solution on an IBM 7094 computer, compared to 12 minutes by Cord on an IBM 7070. Weingartner's solution also produced an exact solution, where Cord's, using the Lagrange multiplier formulation did not produce the optimum. Cord attributed the less than optimal solution to the coarseness of the iterations on the Lagrange multiplier, but Weingartner concludes from a comparison with the exact solution and a finer evaluation of the Lagrange multiplier formulation, that the exact solution can not be found using the Lagrange multiplier formulation.

4.4 Conclusions on Existing Capital Budgeting Formulations

The capital budgeting models discussed in this chapter were investigated for the purposes of finding a suitable model for selecting the optimal course of action in the facility maintenance planning problem. In each of the above models, the timing of the funding is determined in advance, and the question answered is which projects should be funded. In the facilities maintenance planning problem, the solution must also indicate when funding of a project should be accomplished.

The linear and integer programming models provided the option of several approaches to expressing dependence between projects. This is an advantage over the dynamic programming model which rapidly becomes dimensionally intractable when constraint conditions are considered. In Chapter V a dynamic programming algorithm is developed which treats the time of initial funding problem inherent in the planning problem; and reasons for the selection of dynamic programming will be presented.

CHAPTER V

APPLICATION OF DYNAMIC PROGRAMMING TO FACILITIES MAINTENANCE PLANNING

Chapter II introduced the systems orientation of facility maintenance planning for a system of military bases. In Chapter III a value system for planning was formulated using a value system design process as a model. Chapter IV provided a review of mathematical programming techniques used in capital budgeting and resource allocation problems. In this chapter the objective function of equation (3-7) is transformed into a one state variable dynamic programming recursion equation. This formulation permits the evaluation of the feasible combinations for funding a set of repair projects, considering the funding of each project in every year of a budget horizon. Practical limitations are introduced and methods for reducing the effects of these limitations are described. The results of a computer solution to a sample problem are provided.

5.1 Transformation of the Value System Objective Function to a Dynamic Programming Recursion Equation

5.11 The Project Return Function, $g_i(q)$

The dynamic programming recursion equation (4-6) requires that for every feasible value of resource as-

signed to an activity, there must be defined some return function. The general form of equation (3-7) considered for a single project has this characteristic.[1] The resource required is the present worth of the cost of the repair when it is funded in year k. Where k = 1,2, 3, . . , p, as k varies, the net present worth return varies discretely, thus, a return function is generated. Given project i:

a. $$g_i(q) = \sum_{j=1}^{n} c_{ij}a_j - \sum_{j=1}^{k} c_j a_j + Ra_k + \sum_{j=k}^{n} c'_j a_j \qquad 5\text{-}1$$

$$1 \leqq k \leqq p$$

b. $q = Ra_k$

$$1 \leqq k \leqq p$$

k is integral

5.12 The Recursion Equation

Equation (3-7) can now be transformed into a recursion equation:

$$f_n(q) = \mathrm{MAX}\left|g_n(q_n) + f_{n-1}(q - q_n)\right| \qquad 5\text{-}2$$

$0 \leqq q \leqq Q$ for feasible values of q

Q = the present worth of all budgets before the budget planning horizon,p.

5.2 Practical Application of the Model

The dyanmic programming recursion equation (5-1)

for evaluating the objective function equation (3-6) has the following desirable characteristics:

1. The optimal time of funding each project in a set of projects may be determined for a range of resource allocations.
2. The single constraint on the present worth value of future annual budgets defines an optimal set of annual budgets.
3. The optimal time of funding each project independently is evaluated when the quantity of resource available, Q, is a set at a level which would permit immediate funding of all projects.
4. A set of solutions are generated which establish a cost/effectiveness comparison for a range of feasible resource investment values.

The assumption of one state variable, Q, and independence of projects creates the following disadvantages:

1. Dependence of relations between projects are excluded.
2. The model is deterministic.
3. Large sets of projects make the solution intractable.
4. Annual budgets imposed by higher authority can not be modelled.

All of the above shortcomings of the basic equation are caused by the rapid increase in dimensionality when more

than one state variable is specified.

For the large military facility system, the number of repair projects to consider each year can be recognized to be in the hundreds, if each base submits only a single project. Consideration of all projects over a ten year planning horizon raises this to thousands of projects. A practical solution requires some method of reducing the dimensionality of the problem. Two methods have been suggested in the literature: "coarse grid" search (2); and decomposition (5). A form of each technique has been applied in the computer program developed utilizing equation (5-2).

5.21 Coarse Grid Search

In using a coarse grid search the number of feasible values of the state variable, q, searched as q varies from zero to Q is reduced by sampling the feasible values at equal intervals. A coarse grid problem solution can be used to define a local neighborhood of feasible values to be searched exhaustively. Bellman recommends the method for return functions, $g_n(q)$, not subject to sharp peaks which may be bracketed by the grid interval.

5.22 Decomposition

The technique of decomposition was suggested by Dantzig for linear programming problems. In cases where project independence is assumed, and a single state

variable is used, its use in dynamic programiing appears feasible. Decomposition refers to decomposing a large set of projects into a set of subsets of projects to be solved independently. After each subset has been solved by application of dynamic programming, the solutions of each subset are treated as return functions in an aggregation application of dynamic programming.

5.23 Reduction in Number of Feasible Values

A third method of reducing dimensionality is to reducethe total number of feasible values in the return function $g_n(q_n)$ of equation (5-1). This may be done by shortening the planning horizon. In the fiscal sense only the current budget year is important, for plans for future years can and will be changed as the environment changes. Therefore, if a shortened planning horizon provides an equivalent funding plan in terms of total resources to be expended, and total savings generated, it may be assumed that the shortened horizon search is equivalent to the long horizon search.

5.3 Computer Program

A computer program was written in FORTRAN IV for use on the IBM 360 model 75 computer. The program was developed to evaluate the methods of increasing the feasible problem size discussed above. The comparisons made were:

1. A comparison of a coarse grid search on the state variable with a search of all feasible values.
2. A comparison of a coarse grid search in the state variable with a search of all feasible values on subsets of projects aggregated by dynamic programming. A coarse grid sampling of the solution of the subsets is used as a return function $g_i(q)$.
3. A comparison of three values of the budget horizon ranging from five to twenty years.

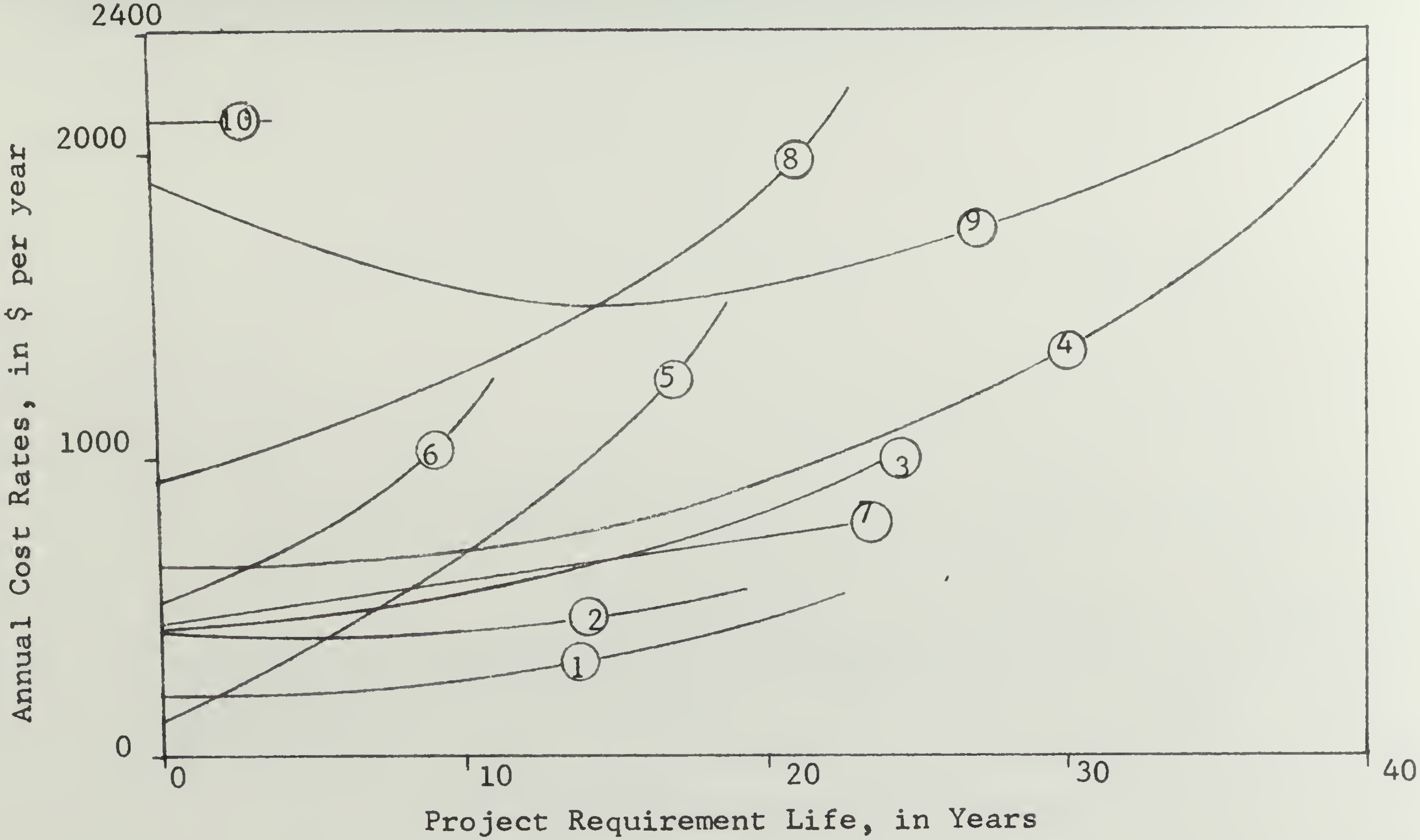
5.31 Data

The data for the program consists of a set of fifty projects. Figures 8 and 9 represent the original annual cost streams c_{ij} and revised annual cost stream c'_{ij} for a basic set of ten projects. Discrete annual cost values were interpolated from these continuous representations. Figure 10a lists the set of repair costs, R_i, for the basic ten projects, and Figure 10b is the set of discount factors r_i for the set of fifty projects. The annual costs and repair costs are repeated five times to construct the fifty project set. Each project has a unique discount rate, r_i, which gives it a unique return function.

5.32 General Description of the Program

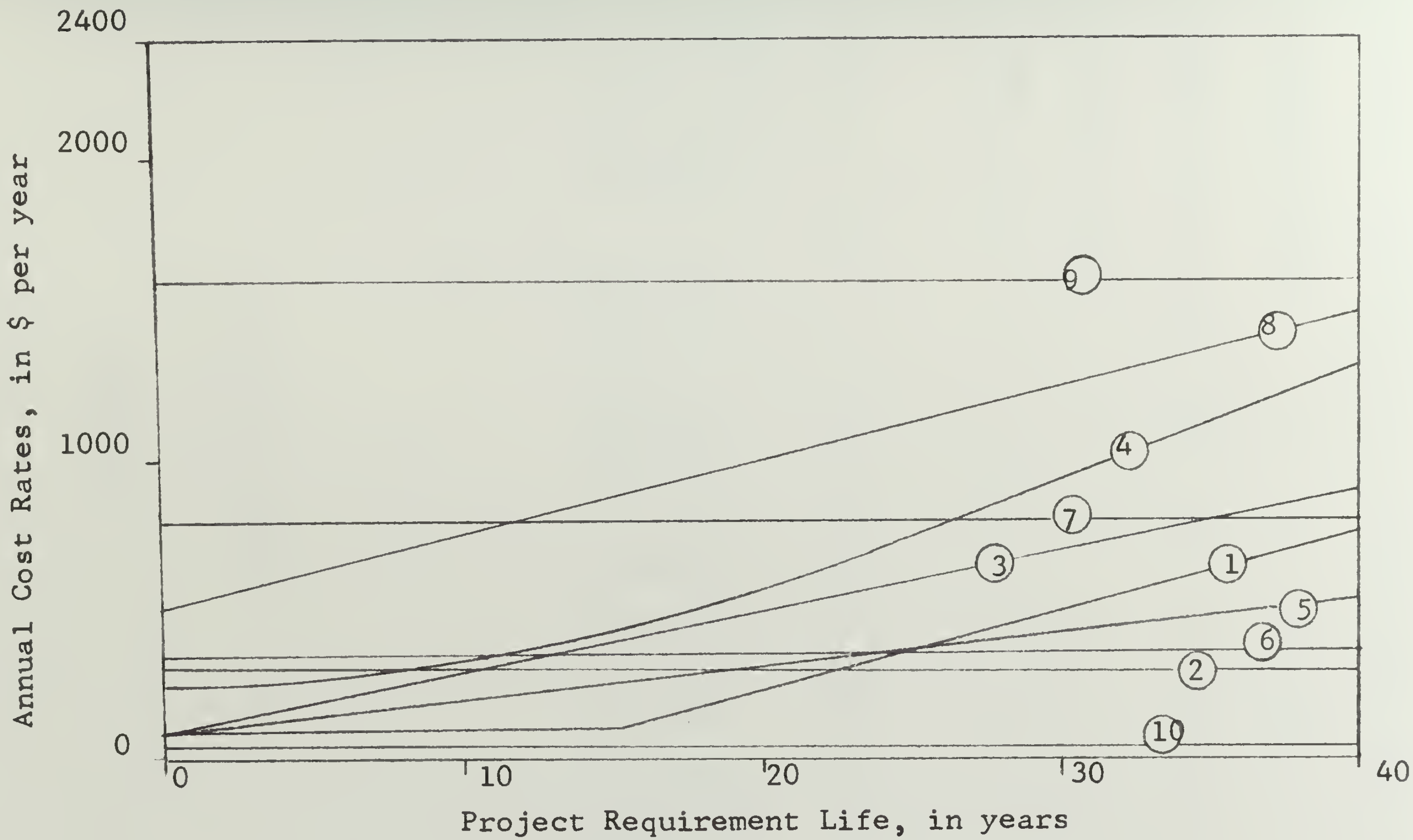
The program consists of these major elements:

1. Return function generator.



ANNUAL COST STREAMS FOR BASIC 10 PROJECTS, BEFORE REPAIRS

FIGURE 8



ANNUAL COST STREAMS FOR BASIC 10 PROJECTS--REPAIRS FUNDED AT TIME ZERO
FIGURE 9

a. Project Repair Costs, R_i, for 10 Basic Projects

Project Number	1	2	3	4	5
Repair Cost, in \$	270	200	400	1600	1900
Project Number	6	7	8	9	10
Repair Cost, in \$	250	800	1200	1180	2500

b. Discount Rates, r_1, for Each of 50 Projects

Projects	1	2	3	4	5	6	7	8	9	10
1--10	.06	.07	.10	.15	.05	.08	.20	.25	.22	.30
11--20	.04	.06	.08	.13	.08	.13	.15	.20	.16	.35
21--30	.02	.05	.07	.10	.06	.12	.10	.18	.10	.25
31--40	.01	.04	.04	.08	.04	.10	.08	.15	.08	.10
41--50	.05	.03	.02	.03	.02	.08	.04	.10	.04	.05

PROJECT PARAMETERS

Figure 10

2. Dynamic programming subroutine.

3. Data storage and output section.

The principal features of these elements will be outlined. The computer listing and an interpretation of the program symbols is included in the appendix.

Return Function Generator

The return function generator operates on equation (5-1). The sets of annual costs, c_{ij}, and c'_{ij}; the length of the planning horizon, p; the maximum project life, n; and the set of repair costs, R_i, are provided as inputs. Using this data, the stage return function $g_i(q)$, is computed for all feasible values. There is one feasible value for each period in the budget horizon. A penalty cost is included in the program to be assigned when the allocation of the resource is proposed after the project has terminated. This is detected when a value of zero is assigned to c_{ij}. If an intermittent annual cost of zero exists in a project, the assignment of a nominal cost other than zero will avoid the penalty cost. The set of all return functions $g_i(q)$ is stored for use by the dynamic programming subroutine.

Dynamic Programming Subroutine

This subroutine operates on equation (5-2) for discrete values of q. Three features are of interest:

1. The state variable is incremented over all feasi-

[illegible]

ble values by first incrementing over the feasible values q_n of the stage return function $g_n(q_n)$ and when these are exhausted, adding successively to the highest value of q_n, the values of q form the previous stage. This permits every feasible value from the lowest feasible value of the present return function, q_n, to a value which permits all projects considered to be funded in the year generating their independent optimal returns.

2. The optimum values of q_n and $(q - q_n)$ are found by varying q_n ovef all feasible values of the return function, $g_n(q_n)$, or to the limit of the state variable.
3. When a coarse grid is applied to the state variable, the only modification to the program is to increment the first values of q_n by an interval greater than one. The values of q from the previous stage applied afterward, already exhibit the greater interval.

Data Storage and Output Section

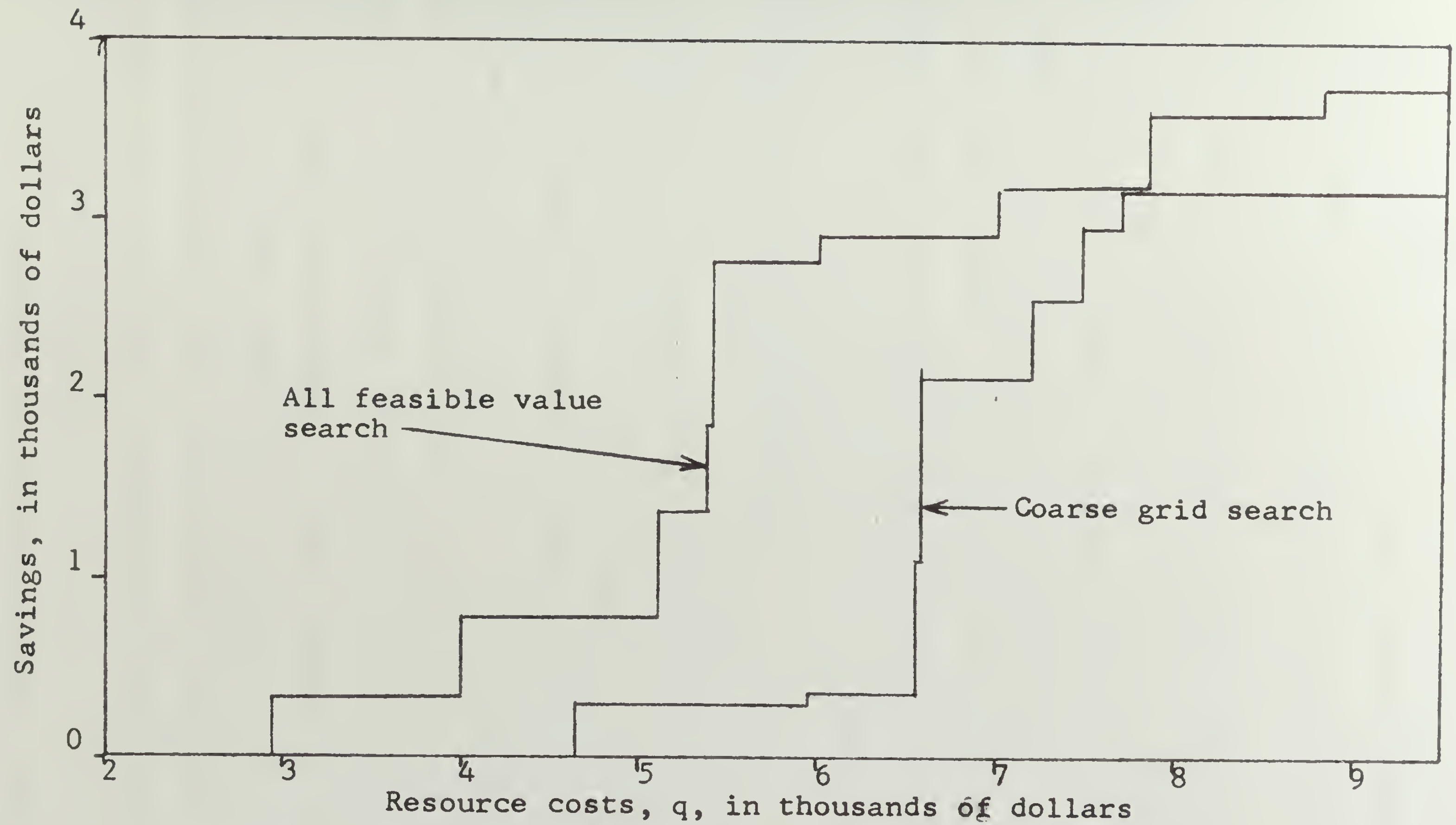
The principal item of interest in this section of the program is the method of generating a synthetic return function for the application of the subroutine to the aggregation step in the decomposition method. The method used is coarse grid sampling. The results of each solution of a subset is a set of decisions

defining the funding year of each project for a set of savings and resource costs for each feasible increment of resource allocation, q. The technique used is to sample the savings, $f_n(q)$, and cost increments, q, with an interval size that will permit the total set of increments to be represented by twenty values. For the selected increments, the cost, q, and savings, $f_n(q)$, values and the funding year decisions for the projects associated with these values are stored to be retrieved if that increment is used in the final aggregated solution.

5.4 Results of the Computer Evaluations

5.41 Coarse Grid Versus Exhaustive Search of the State Variable Comparison

A ten project, twenty year planning horizon problem was solved using both complete search of feasible values of q and a coarse grid search where only one third of the feasible values of the state variable were searched. Figure 11 shows a plot of cost, q, and return $f_n(q)$ functions. The coarse grid solution failed to reach the level of return for equivalent costs found by the complete search process. It is reasonable to infer that the coarse grid solution failed to discover efficient solutions in the early stages and therefore, previous stage solutions could not be optimal. An exhaustive



COARSE GRID VERSUS ALL FEASIBLE VALUE SEARCH OF THE STATE VARIABLE

Figure 11

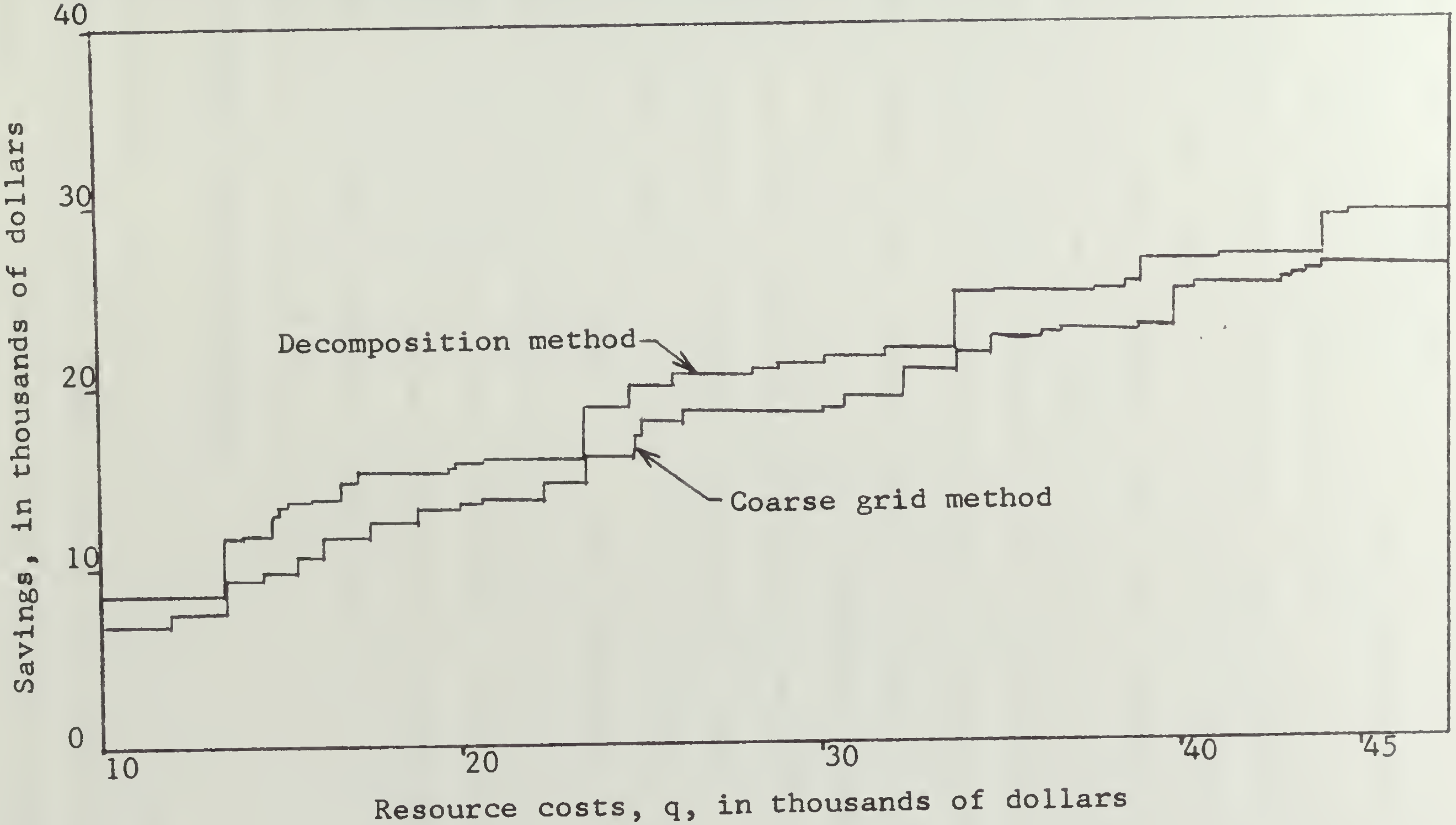
search (not performed) of the local neighborhoods defined by each project may have produced identical solutions. The comparison made in the following section suggests that large project sets may approach the optimal solution with greater accuracy.

The shape of the cost versus savings curve, $q/f_n(q)$, is of interest, however, for it defines the efficiency of the resource. As may be expected, the rate of increase of saving for the lower values of resource cost is greater than the final values. This would be beneficial in defining trade off values between facilities maintenance programs and other programs competing for scarce dollars.

5.42 Application of Decomposition

A comparison was made of the solution of a set of fifty projects with a twenty year budget horizon using a coarse grid search of the state variable considering one third of the feasible values in one case, and the decomposition method in the second case. In the decomposition method, the fifty projects were divided into five subsets, subjected to the dynamic programming routine using complete search of the state variable. Twenty samples of each subset solution were taken, and the dynamic programming routine was applied to the five synthetic projects composed of sample cost and return increments of the subsets. The results are shown in Figure 12.

The plot of cost versus savings for each method



DECOMPOSITION SOLUTION VERSUS COARSE GRID SEARCH OF STATE VARIABLE

Figure 12

shows a closer correlation than in the previous ten project problem, however, the decomposition method provides consistantly higher returns. Another important consideration is the computational time required. The coarse grid method requuired approximately seven minutes of computation time compared to approximately three minutes for the decomposition method. Based upon these factors, the decomposition method appears superior to the coarse grid method. There is, however, still the question of the efficiency of decomposition versus a complete state variable search using a single dynamic programming solution. This method was not attempted because it would require more rapid memory space than was available on the IBM 360 model 75 computer configuration used.

A useful feature of the dynamic programming formulation indicated in Figure 12 is the reporting of alternative solutions providing approximately equal returns within a small range of cost difference. This would be helpful in justifying the selection of contingent projects or other constraint conditions which were not included in the problem formulation.

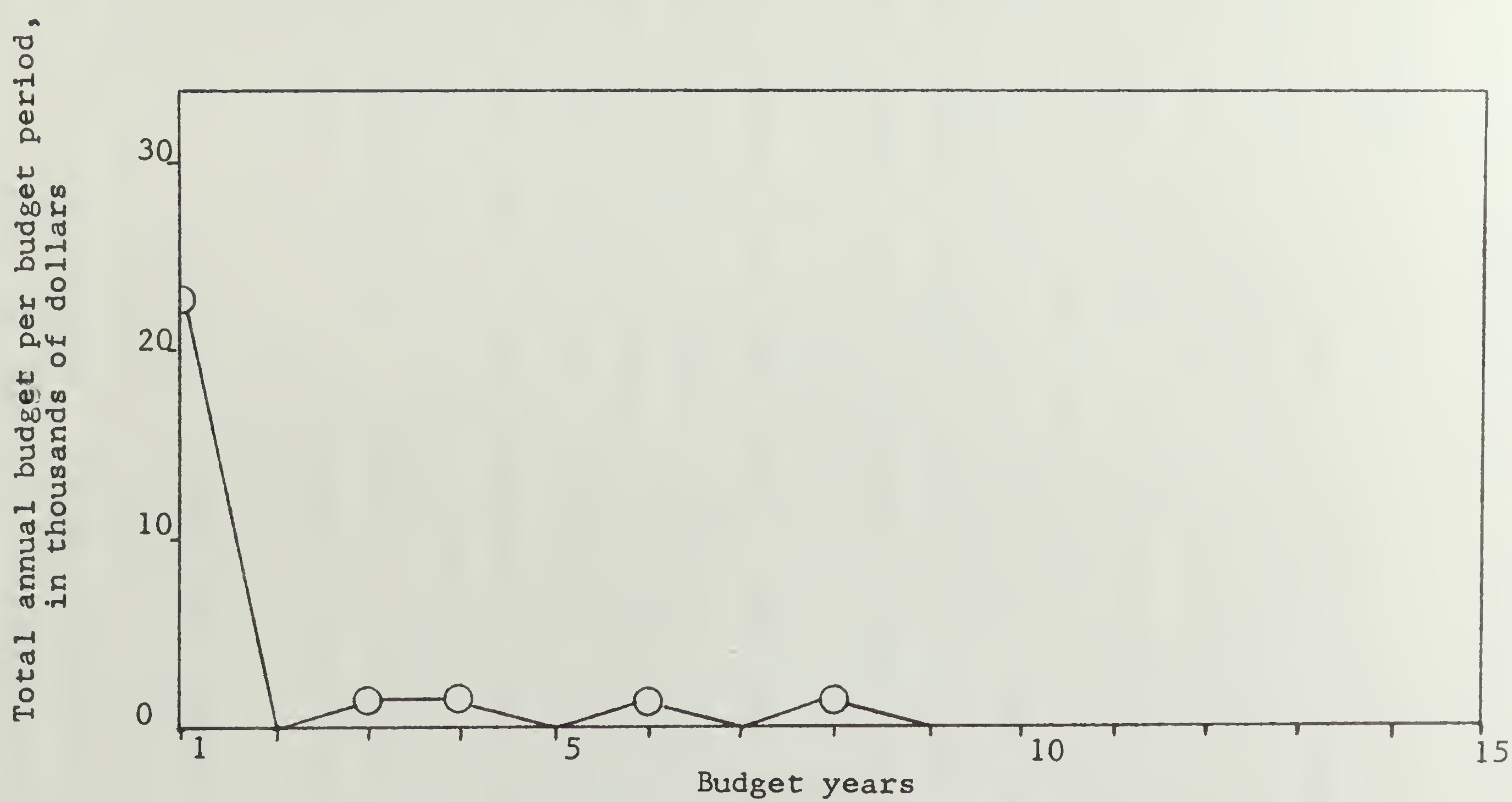
5.43 Annual Budget Comparison

An attempt was made to compare the distribution of projects within annual budget periods for several lengths of budget horizon to determine whether or not a short

budget horizon would provide equivalent selection of projects for comparable uniform annual budgets. The program, as formulated, did not provide logical output data when the budget horizon was reduced below a twenty year design. Figure 13 provides an indication of the results that may have been expected, however. Figure 13 is a plot of annual budget costs versus budget periods. The present worth value of total project and costs is $28,322 which is approximately equivalent to $2500 per year for twenty years at 6% interest. The majority of projects are shown to be funded in the first budget year, and all of the projects in the three other budget periods. This seems to indicate that the dynamic programming formulation does not distribute cost uniformly. This result may also have been due to the return function for the set of projects treated. There seems to be sufficient evidence to conclude that the constrained solution for a long budget horizon is not a practical approach. The decision maker must still determine which projects he will fund in the current budget year.

Footnote:
1. Return function relation suggested by Professor J. M. English.

[illegible]



OPTIMAL ANNUAL BUDGET DISTRIBUTION

Figure 13

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER RESEARCH

6.1 Conclusions

The problem of maintenance planning for facility complexes has been recognized to be a management subsystem which must operate within the constraints of the primary systems it serves. Facility maintenance must be kept at condition levels which permit the missions which they support to function adequately. This conflicts with a maintenance management objective of minimum cost. It has been argued that an objective function for maintenance planning must be based upon a return function which assigns penalties when facility conditions infringe upon the constraints defining acceptable conditions prescribed by the primary missions served by facilities.

The general value system developed for planning maintenance was found to be impractical at the present, because of the lack of methods of determining values other than economic values. Methods are available, but additional research will be necessary before they may be applied to facilities planning.

If it were assumed that all pertinent values could be measured, there is still the problem of solving the resulting objective function. The dimensionality problem becomes unmanageable when only a limited number

[illegible]

of constraints and variables are present. The dynamic programming algorithm employing a single state variable and a decomposition process appears to offer a tractable solution for sets of projects up to one thousand; however, additional research on the efficiency of the technique would be required. While tractable, the solution generated may not be useful for budget planning. Irregular annual budget patterns are generally incompatible with financial policies based on uniform budget for facility maintenance. This irregularity is a function of the project inputs, not the solution technique. Projects in Chapter V yielded maximum returns in the first year as defined. Thus, the set of projects was biased toward immediate funding. This type of biasing may be due to a project originator's attempt to optimize his portion of the system, but may also reflect the state of the total system. If the system were previously undermaintained, immediate funding would be the expected optimal solution. Conversely optimal maintenance of a completely new system should call for heavy funding in the distant future. By proper review procedures biasing by project originators can be reduced. However, the problem of system condition imbalance can not be eliminated. Assuming static conditions and true representation of conditions, the irregular budget pattern represents the optimal funding requirements. Unfortunately, this is seldom feasible and a solution

constrained to approximately uniform annual budgets is still required.

6.2 Recommended Future Research

It is estimated that maintenance projects for a large facility system such as the military base system will number in the hundreds for only a five year projection. This fact, plus the need for introducing budget constraints and project dependency relations appears to rule out methods of linear, integer and dynamic programming, because of the limited capabilities of the current generation of computers. One alternative approach which may show promise is the method of statistical sampling using biasing rules. Arcus[1] applied this approach to the assembly line balancing problem and was successful in finding optimal solutions under constraint conditions to problems involving up to one thousand tasks. The assembly line problem appears to be analogous to the maintenance planning problem in its magnitude and requirement for accounting for constraints. This method is suggested as a fruitful area for additional research.

Footnotes:

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[illegible]

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APPENDIX A

PRINCIPAL COMPUTER PROGRAM TERMS

1. EAC A three dimension array provided as input which provides the annual costs for each project, c_{ij}, for the arbitrary project life horizon of n years (I YR).
2. EAACI A three dimensional array equivalent to the annual cost for each project, c'_{ij}, where the repair, R_i, is assumed to be funded immediately.
3. RN An input vector of the repair costs, R_i, for each project.
4. BIR The interest rate on all budget money.
5. CIR An input vector of the discount rates for each project, r_i.
6. LT The number of projects in a subset to be optimized when the solutions of the subsets of projects are treated as return functions.
7. NS The number of subsets of size LT to be optimized when the solutions of the subsets of projects are treated as return functions.
8. JB The number of years in the budget planning horizon, p.
9. RR The set of return functions generated for each project as a function of EAC, EAACI, JB, BIR, IYR.

10. GRRACN The set of cost and return functions for each feasible value of the state variable in the stage being evaluated. It includes q and f(q) for $0 = q = Q$.

11. GRRACO The set of cost and return functions for the previous stage, q_{n-1} and $f(q_{n-1})$.

12. IDEC An array which stores the decisions from each stage and for each feasible value of q. The decision stored for each project is the number indicating the year in which the project is to be funded.

13. RRS An array which stores the final values of GRRACN for each subset of projects. It is converted to RR when the dynamic programming subroutine is used to aggregate the solutions to the subsets of projects.

14. IDECS An array which stores the funding year decisions for each project for each value stored in RRS. It is used to convert the final combined dynamic programming solution to decisions on funding each project.

15. ANCOST A vector of total annual costs for each year in the budget horizon. It is computed for each final value of the state variable. These costs represent recommended annual budgets for optimal returns.

[illegible]

16. JQT The maximum feasible number of increments in each stage.

APPENDIX B

PROGRAM LISTING "REPAIR PROJECTS ALLOCATION PLAN"

```
CCCCCC Clears array IDEC

    DO658IA=1,LT,1
    DO659B=1,JB,1
    DO660IE=1,NS,1
    IDECS(IA,IB,IE)=JB
660 CONTINUE
659 CONTINUE
658 CONTINUE
CCCCCC Initiates generation of return functions for each project
   LS = Project subset number
    DO199LS=1,NS,1
CCCCCC L = project number with a subset of projects
    DO200L=1,LT,1
CCCCCC SUMI is the present worth cost of the future cost stream
   of a project if no repairs are funded.
    SUMI=0.0
    DO50NI=1,40,1
    ANI=NI
    DCII=1.0/(1.0+CIR(L,LS) )**ANI
 50 SUMI=EAC(NI,L,LS)*DCII+SUMI
CCCCCC K increments over the years in the budget horizon

    DO150K=1,JB,1
    AK=K-1
    DCI=1.0/((1.0+BIR)**AK)
    SUM0.0
CCCCCC N increments over the years of project life having annual costs

    DO100N=1,1YR,1
    M=N-K
    AN=N
    DC=1.0/((1.0+CIR(L,LS) )**(AN) )
    IF(M.GT.0)   GOTO 105
    IF(EAC(N,L,LS).EQ.0.0)   GO TO 120
```

```
      SUM=SUM+EAC(N,L,LS)*DC
      GO TO100
  105 IF( K.EQ.JB)    GO TO 130
      SUM=SUM+EACCI(N,L,LS)*DC
      GO TO 100
CCCCCC A penalty cost of $20,000,000 is assessed if funding is considered
     after the life of the project.
  120 SUM=20000000.0
      RR(1,K,L,LS)=RN(L,LS)*DCI
      RR(2,K,L,LS)=-SUM
      GO TO 150
  130 SUM=SUM+EAC(N,L,LS)*DC
  100 CONTINUE
      IF( K.EQ.JB)    GO  TO   140
      SUM=SUM+RN(L,LS)*DCI
CCCCCC SUM is the present worth cost of a revised cost stream.  SUMI -SUM
     is the savings generated
      RR(2,K,L,LS) =SUMI-SUM
      RR(1,K,L,LS) =RN(L,LS)*DCI
      GO TO 150
  140 RR(2,K,L,LS)=0.0
      RR(1,K,L,LS)=0.0
  150 CONTINUE
  200 CONTINUE
CCCCCC  The dynamic programming subroutine is called when the, return
     functions for a subset of projects is complete.
      CALL DYNPG
      JQ=JQT
  613 IF(GRRACO(2,JQ).GT.0.0)   GO TO 612
      JQ=JQ-1
      GO TO 613
  612 JOR=JQ
      JER=JOR/JB
      DO602KE=1,JB,1
CCCCCC  The returns from each application of the subroutine are sampled, the sample
```

```
      returns from each subset are stored as RRS
      JRR=JOR-KE*JER+JER
      IF(KE,EQ,JB)   GO TO 614
      RRS(1,KE,LS)=GRRACO(2,JRR)
      RRS(2,KE,LS)=GRRACO(1,JRR)
      GO TO 615
  614 RRS(1,JB,LS)=0.0
      RRS(2,JB,LS)=0.0
CCCCCC  The related decisions are stored as IDECS
  615 DO603B=1,LT,1
      IDECS(LB,KE,LS)=IDEC(LB,JRR,LT)
  603 CONTINUE
  602 CONTINUE
  199 CONTINUE
CCCCCC  When all subsets of the projects have been optimized, the sample returns
      are converted to variables RR used in the subroutine.
      DO604NL=1,JB,1
      DO605LM=1,NS,1
      RR(2,NL,LM,NS+L)= RRS(2,NL,LM)
      RR(1,NL,LM,NS+1) = RRS(1,NL,LM)
  605 CONTINUE
  604 CONTINUE
CCCCCC  The variable LT, the number of projects, is stored as LTO.  LT is
      then used to identify subsets treated as projects.
      LTO=LT
      LT=NS
      LS=NS+1
CCCCCC  The dynamic programming subroutine is called after all inputs have been
      revised to comply with the subroutine variable names
      CALL DYNPG
      JQ=JQT
CCCCCC  The results of the aggregation process are sampled, establishing
      a coarse grid print out index JOR
  635 IF(GRRACO(2,JQ).GT.0.0)  GP TP 617
      JQ=JQ-1
```

```
    GO TO 635
617 JOR=JQ
    JORI=JOR
CCCCCC  The print out format is established.

628 WRITE(6,618)
618 FORMAT(T10,'COST',T40,'SAVINGS')
    WRITE(6,619) GRRACO(2,JOR),GRRACO(1,JOR)
CCCCCC The variable for annual cost is cleared

    DO623KB=1,JB,1
    ANCOST(KB)=0.0
623 CONTINUE
CCCCCC The decisions associated with the return are located and
     converted to scaler variables for print out
    DO606LS=1,LT,  1
    KE=IDEC(LS,JOR,LT)
    LTTO=LTO-1
    DO607LB=1,LTTO,2
    KSOL=IDECS(LS,KE,LS)
    NOPRI=LS*10*LB-10
    COSTI=RN(LB,LS)
    PRECO1=RR(1,KSOL,LB,LS)
    PRESAI=RR(2,KSOL,LB,LS)
    KSOL2=IDECS(LB+1,KE,LS)
    NOPR2=LS*10+LB-9
    COST2=RN(LB+1,LS)
    PRECO2=RR(1,KSOL2,LB+1 ,LS)
    PRESA2=RR(2,KSOL2,LB+1,LS)
CCCCCC  The decision data for each project is printed out

    WRITE(6,622)NOPR1,KSOL,PRECO1,COST1,PRESA1,NOPR2,KSOL2,PRECO2,
   CCOST2,PRESA2
622 FORMAT(1H0,T10,I4,T15,I4,T20,F14.0,T34,F14.0,T48,F14.0,T65,I4,T70
   C,I4,T80,F14.0,T99,F14.0,T113,F14.0)
```

```
      ANCOST(KSOL) =ANCOST(KSOL)+COST1
      ANCOST(KSOL2)=ANCOST(KSOL2)+COST2
  607 CONTINUE
  606 CONTINUE
CCCCCC  The total annual budget costs are printed out

      WRITE(6,625)
  625 FORMAT(1HO,T10,'ANNUAL COSTS-NOT DISCOUNTED' )
      JBZ=JB-1
      DO624KB=1,JBZ,4
      WRITE(6,626)ANCOST(KB),ANCOST(KB+1),ANCOST(KB+2),ANCOST(KB+3)
  626 FORMAT(1HO,T10,F14.0,T30,F14.0,F 14.0,T70,F14.0)
  624 CONTINUE
      JOR+JOR-3
CCCCCC  JOR is incremented and tested for completion of the
      sample print out
      IF(JORI-JOR.LE.50)  GO TO 628
  627 RETURN
      END
CCCCCC  The variables in the problem are dimensioned.  The funding year for each
     project is stored as an integer to save storage space.

      INTEGER*2 IDEC(10,211,10), IDECS(10,42, 5)
      COMMON  IDEC           ,IDECS    ,     CIR(10,5),EAC(40,10,5),
     CEAACI(40,10,5),GRRACO(2,211),GRRACN(2,211),RN(10,5)RR(2,211,10,6)
     C,LT,JB,BIR,IYR,NS,LS      ,RRS(2,42,6), ANCOST(21)  JQT
  807  READ(5,123)LT,JB,BIR,IYR      ,NS
  123 FORMAT(I6,I6,F6.3,I6,I6)
       READ(5,124)CIR
  124 FORMAT(10F6.3)
       READ(5,125)EAC
  125 FORMAT(10F8.0)
      READ(5,126)EAACI
  126 FORMAT(10F8.0)
```

```
     READ(5,127)RN
127 FORMAT(5F16.0)




    SUBROUTINE DYNPG
CCCCCC The array GRRACO is cleared


    JQT=(JB-1)*LT+1
    DO655ID=1,LT,1
    DO656IC=1,JQT,1
    DO657IG=1,LT,1
    GRRACO(1,IC)=0.0
    GRRACO(2,IC)=0.0
    IDEC(ID,IC,IG)=JB
657CONTINUE
656 CONTINUE
655 CONTINUE
CCCCCC  The number of stages is indexed by L

    DO300L=1,LT,1
665 JQT=L*(JB-1)
666 JO=JB-1
    JN=1
CCCCCC The maximum number of increments of the state variable is defined by JQT
    JQ indexes the state variable.  QI is the value of the state variable.
    DO400JQ=1,JQT,1
    IF(JO.EQ.0)  GO TO 410
421 QI=RR(1,JO,L,LS)
    JO=JO-1
    GO TO 430
```

```
410 QI=RR(1,1,L,LS) +GRRACO(2,JN)
    JN=JN+1
CCCCCC   An initial stage return of $-20,000,000 is established to insure
   the first increment will improve the return
430 TRR=-20000000.0
    GRRACN(2,JQ)=QI
    GRRACN(1,JQ)=TRR
    JOI=JB
     IF(JQ.LT.JB)  GO TO 855
    JBZ=JB
    GO TO 854
855 JBZ=JQ+1
CCCCCC The number of feasible values of the stage return g_n defined as QII
   in the program is indexed.
854 DO500JQI=1,JBZ,1
    IF(JOI.EQ.0)  GO TO 500
520 QII=RR(1,JOI,L,LS)
    IF(L.EQ.1)  GO TO 541
667 JMI=(L-1)*(JB-1)
668 QIIB=QI-QII
CCCCCC   The remainder of the state variable QIIB is used to fund
              the largest value of f_{n-1}.
542 IF(QIIB.GE.GRRACO(2,JMI) ) GO TO 540
    JMI=JMI-1
    IF( JMI.EQ.0)  GO TO 541
    GO TO542
540 TERRC=RR(2,JOI,L,LS) + GRRACO(1,JMI)
    TERRCD=RR(1,JOI,L,LS)+GRRACO(2,JMI)
    LR=JOI
    JRI=JMI
    GO TO580
541 TERRC=RR(1,JOI,L,LS)
    TERRCD= RR(1,JOI,L,LS)
    LR=JOI
669 JRI=LT*(JB-1)+1
```

```
580 JOI=JOI-1
CCCCCC  The new trial value stage return, TERRC is tested against the previously
    established value, TRR
590 IF(TERRC.LT.TRR)  GO TO 500
    TRR=TERRC
    GRRACN(1,JQ)=TRR
    IF(JQ.EQ.1)  GO TO 856
    IF (GRRACN(1,JQ).LT.GRRACN(1,JQ-1)) GO TO 850
CCCCCC  The stage decisions are stored
856 LN=1
591 IDEC(LN,JQ,L)=IDEC(LN,JRI,L-1)
    LN=LN+1
    IF(LN.EQ.(LT+1) )GO TO 592
    GO TO 591
592 IDEC(L,JQ,L)=LR
    GO TO 500
850 LN=1
852 IDEC(LN,JQ,L)=IDEC(LN,JQ-1,L)
    GRRACN(1,JQ)=GRRACN(1,JQ-1)
    LN=LN+1
    IF(LN.EQ.LT+1) GO TO 500
    GO TO 852
500 CONTINUE
400 CONTINUE
CCCCCC  GRRACN VALUes are transferred to the variable GRRACO

    DO202JNIT=1,JQT,1
    GRRACO(1,UNIT) = GRRACN(1,JNIT)
    GRRACO(2,JNIT) = GRRACN(2,JNIT)
202 CONTINUE
300 CONTINUE
CCCCCC  Print out is limited to the final values of the  aggregated solution.

    IF(LS.EQ.NS+1)  GO TO 829
```

```
      GO TO 831
CCCCCC   The project return functions RR, the total returns GRRACO and the funding
      decisions for each project are printed out.
  829 WRITE(6,129)GRRACO
  129 FORMAT(1H,10F13.0 )
      WRITE(6,128)( ( ( RR(I,KZ,LZ ,LS),I=1,2),KZ=1,JB),LZ=1,LT)
  128 FORMAT(1H ,6F20.0)
      WRITE(6,131)((IDEC(LN,JRI,LT),LN=1,LT),JRI=1,110)
  131 FORMAT(1H ,2016)
  831 RETURN
      END


      INTERGER*2 IDEC(10,211,10),  IDECS(10,42, 5)
      COMMON IDEC              ,IDECS    , CIR(10,5),EAC(40,10,5)
      CEAACI(40,10,5),GRRACO(2,211),GRRACN(2,211),RN(10,5),RR(2,211,10,6)
      C,LT,JB,BIR,IYR,NS,LS       ,RRS(2,42,6), ANCOST(21)   ,JQT
```